Mapping Collider Physics to the Night Sky

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Quantum Gravity

- Trying to treat gravity as a field theory runs into problems. To fix them one can

  - Look for UV-complete theory… Strings?

  - Make statements that only rely on the long-distance structure we see?

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \;\Rightarrow\; \underbrace{S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R}_{\text{action formulation}} \;\Rightarrow\; \underbrace{g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}}_{\text{linearization}} \;\Rightarrow \underbrace{\text{Non-renormalizeable}}_{\text{QFT interpretation}}$$

Methodology

- Noether's Theorem tells us

Continuous Symmetries ⇒ Conservation Laws

- In the following we will be looking for a larger set of "physical" symmetries so that

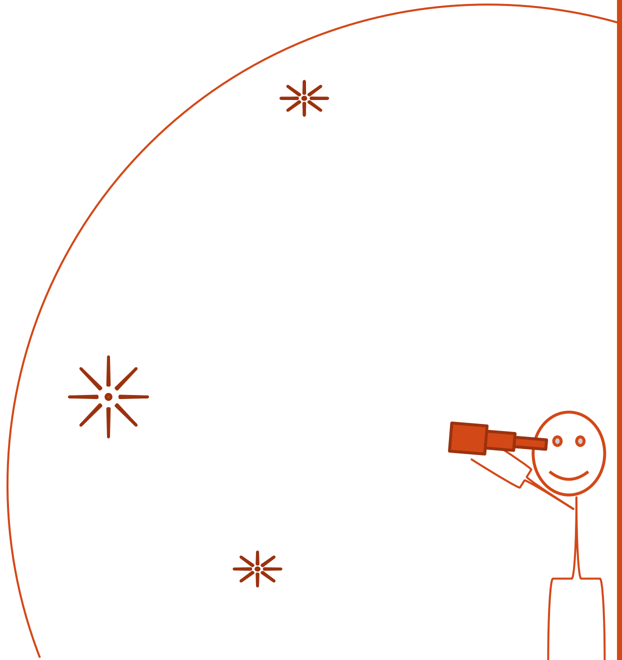
More Symmetries ⇒ More Constraints on $\mathcal{S}$-matrix

A 2D dual to 4D scattering?

The key idea here is…

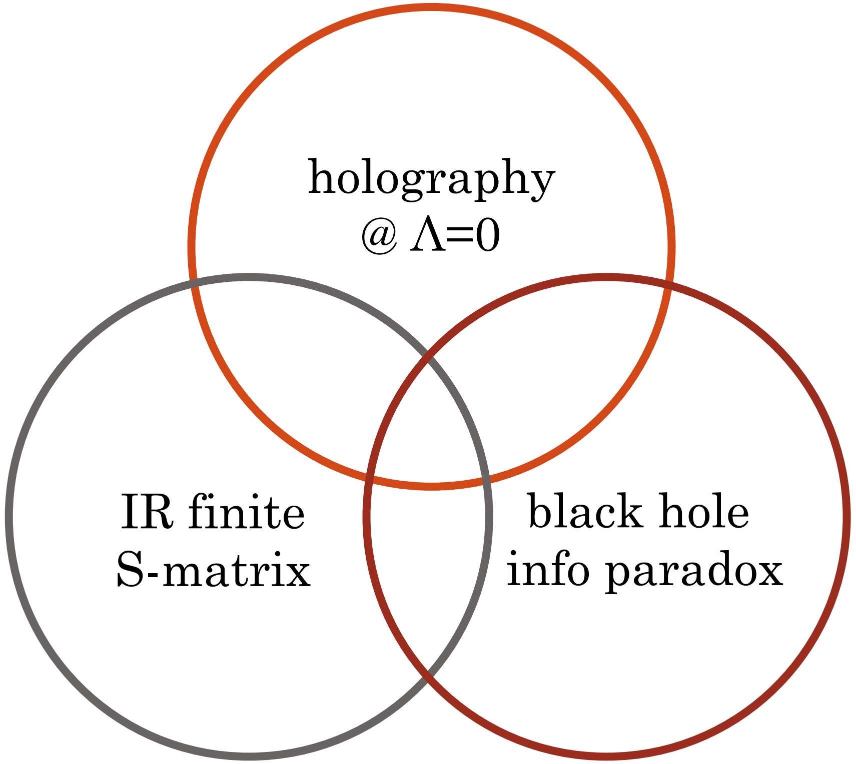
Soft Theorems $\Rightarrow$ **Symmetry Enhancement**

And, that these additional symmetries mimic those of a CFT living on the night sky…



Inspiration

- Today I will be discussing a research program driven by the following three themes…



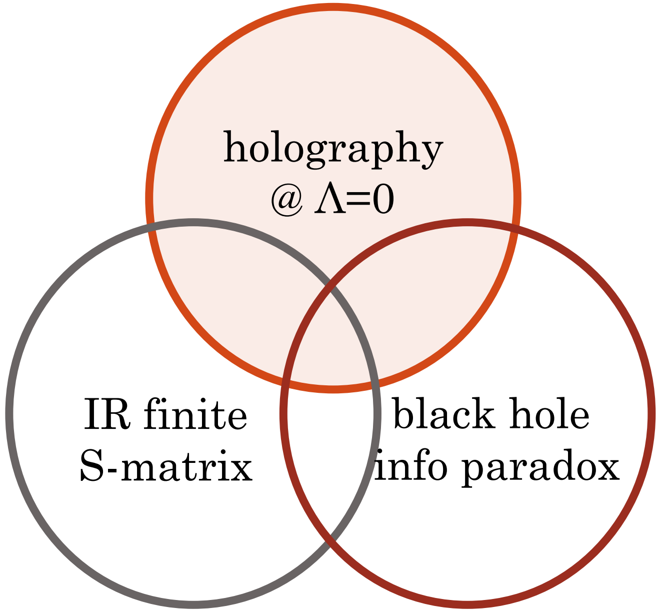
Holography

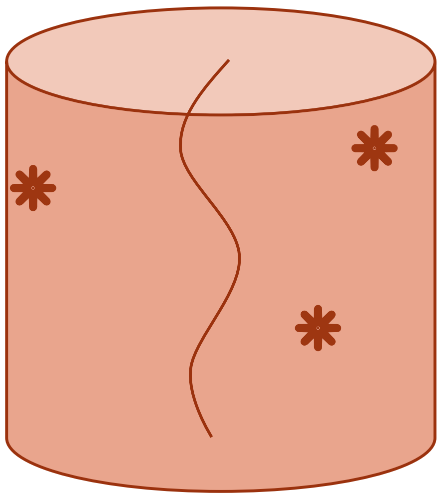
- One of the most fruitful ideas in theoretical physics for the last 20 years has been AdS/CFT

Theory w/ Gravity = Theory w/out Gravity

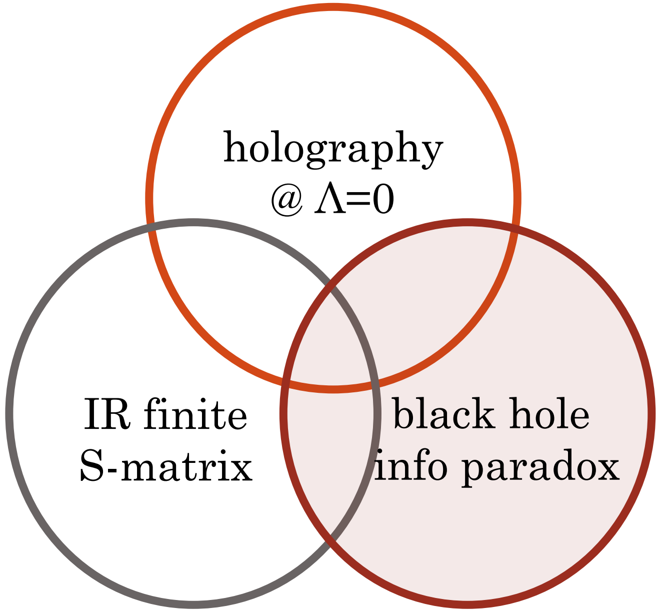
d dim $\Lambda<0$ d-1 dim

- Can we extend this to $\Lambda>0$ how about $\Lambda=0$?



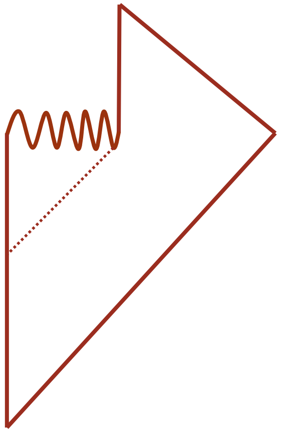


BH Info Paradox



- Once we understand the dual we might be able to describe BH evaporation via the boundary dual – is it unitary?

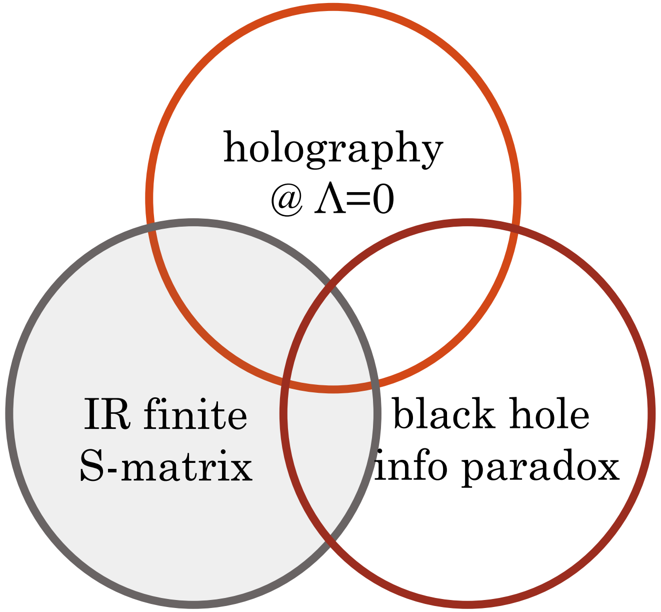
- We will also see that our story involves an infinite symmetry enhancement. What does this say about the no hair theorem?

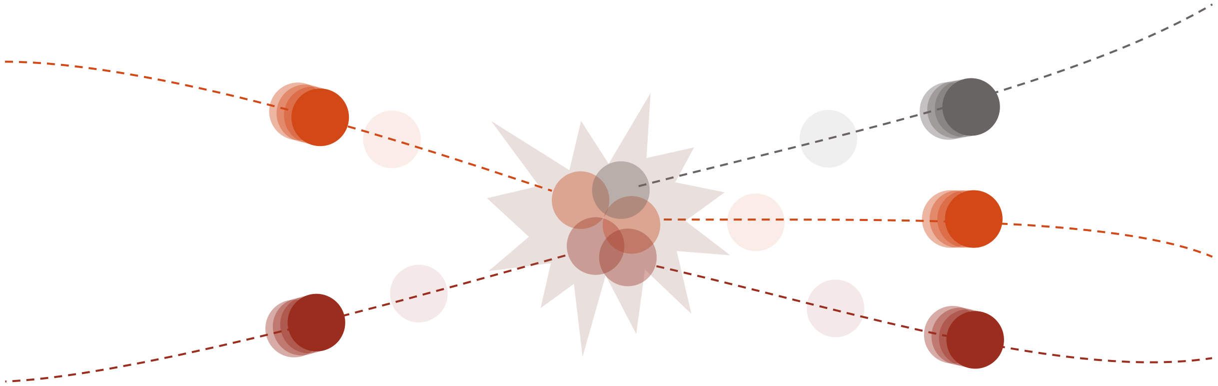


IR divergences

- Our symmetry enhancement is demonstrated by the universality of low energy scattering.

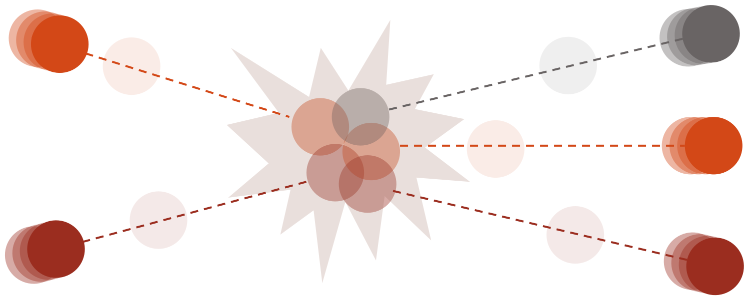
- With this understanding we can reinterpret some IR divergences as symptoms of symmetry non-conservation.





The Setup

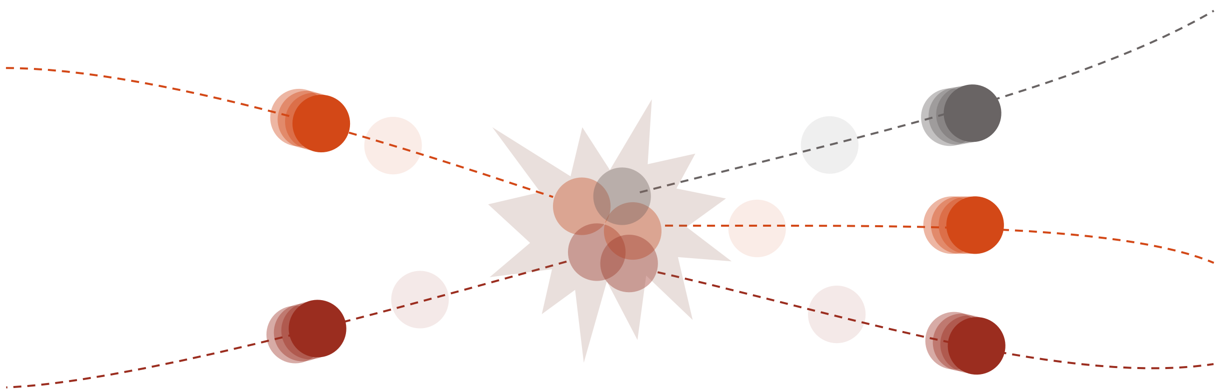
- Let us start by considering a scattering experiment. We prepare particles to collide and observe what comes out.



- If the interactions were short range then at early and late times when the particles are well separated they are free, and it is natural to use momentum eigenstates.

The Setup

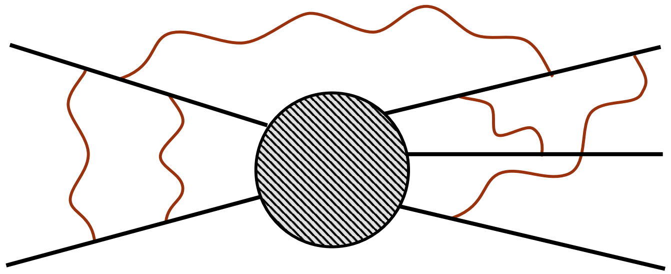
- However the story is not so simple once we have long range interactions.



- The asymptotic trajectories are continually deflected by the presence of other charged particles and the asymptotic states are no longer described by the free Hamiltonian.

Diagrammatica

- If you insist on considering scattering between ordinary Fock states, you get divergences from virtual exchanges between external legs. These exponentiate and cause the matrix element to vanish



- The standard work-around is to recognize that there are some low energy gauge bosons your detector will miss. Thus what you should actually compute is an inclusive observable including soft radiation.

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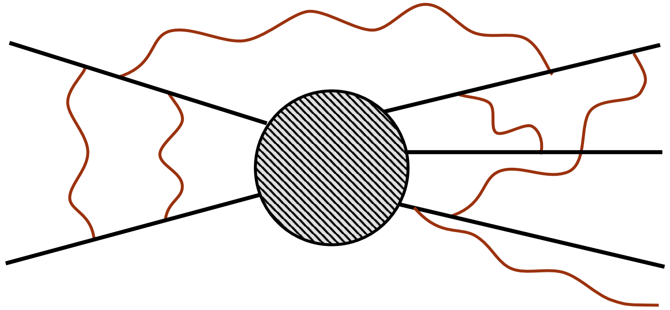

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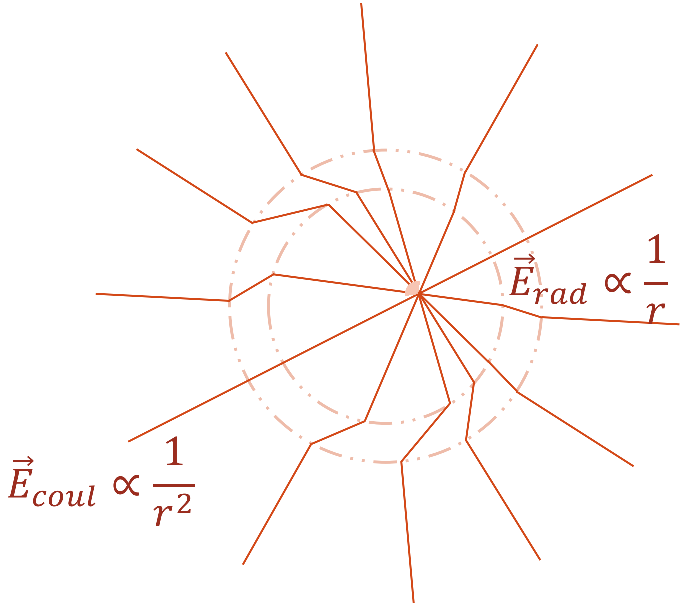
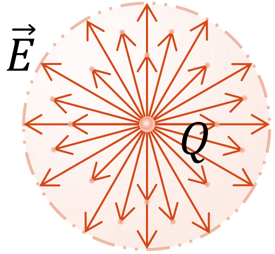
Classical Intuition

- Via Feynman diagrams we've seen that matrix elements with extra soft emissions play an important role. Could we have expected this?
- Accelerating charges radiate so it unsurprising that the chance of scattering between two sets of charged particles with no radiation would generically be zero.



Unsatisfied Constraints

- There are **constraints** on the phase space of radiative gauge bosons and charged particles.
- In particular, certain zero frequency modes of the gauge boson are determined by the charged scatterers.



  - In perturbative QFT computations this corresponds to the universality of **soft theorems**

$$\langle out; \omega\, \hat{q}, \pm | \mathcal{S} | in \rangle = \left(S^{(0)} + \omega S^{(1)}\right) \langle out | \mathcal{S} | in \rangle + \cdots$$

  - While in experiments these modes are referred to as **memory effects**.

$$S^{(0)-} = \sum_k eQ_k \frac{p_k \cdot \epsilon^-}{p_k \cdot q}$$

Large Gauge Symmetries

- As operators, these modes generate gauge transformations which do not die off at infinity.
- We could have started from an asymptotic symmetry analysis

$$A_\mu = A_\mu^{(0)} + \frac{1}{r} A_\mu^{(1)} + \ldots$$

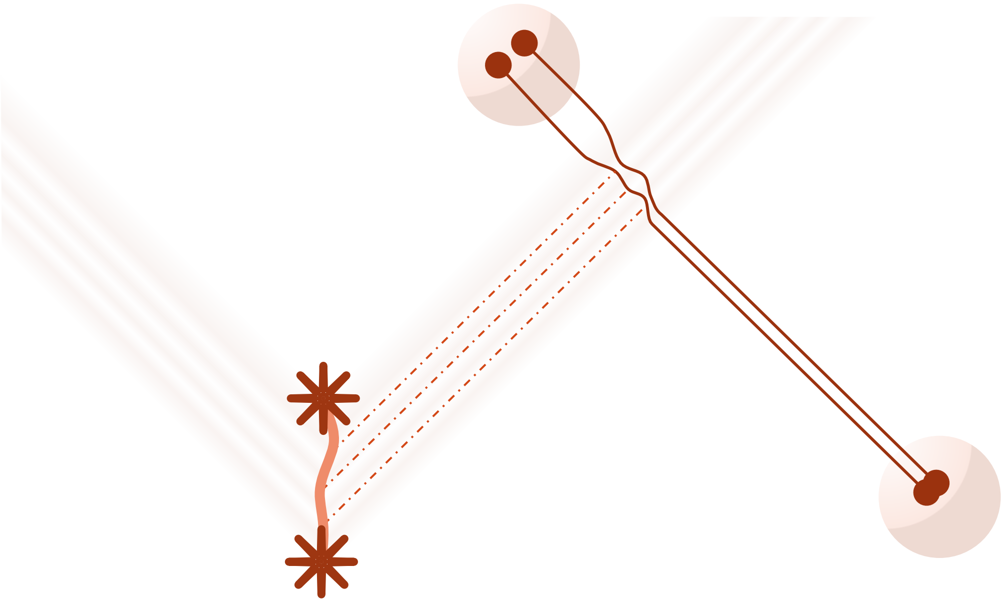
and constructed the charges corresponding to these ‘large’ gauge transformations which preserve our falloff conditions

$$A_\mu \rightarrow A_\mu + \nabla_\mu \lambda$$

Solution Behavior & Free Data

- Don't want boundary conditions to be too restrictive so as to disallow typical scattering processes

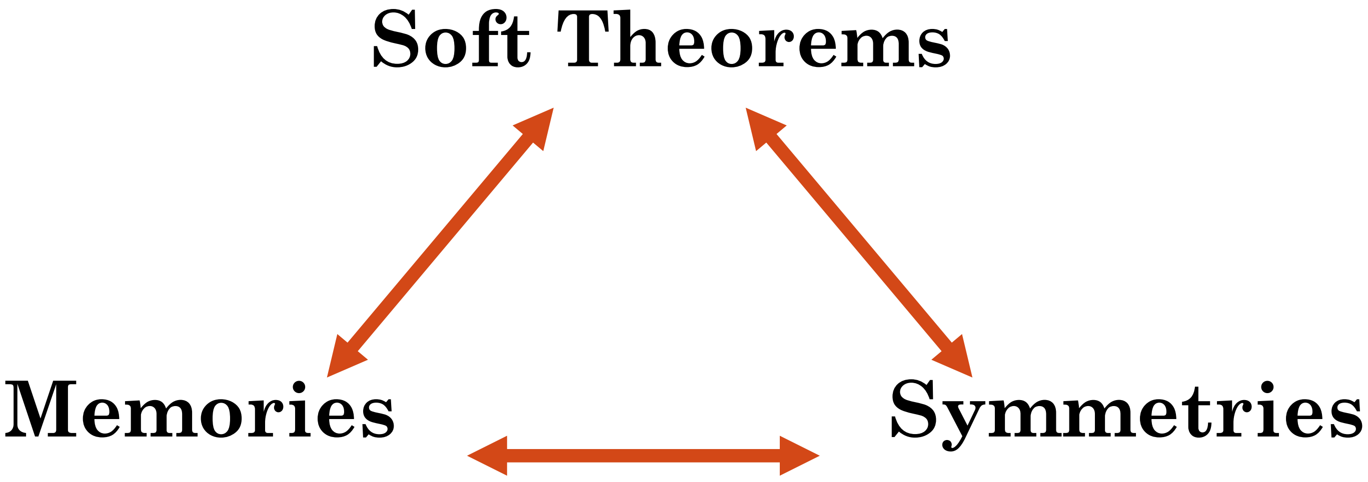
- *Memory effects* are radiation observables, whose values are non-zero in typical scattering processes, and which are related to the sourcing scatterers via constraint equations



A Triangle of Relations

- What can IR physics teach us about gravitational scattering?

There exists a generic pattern of connections between asymptotic symmetries, soft theorems, and memory effects…



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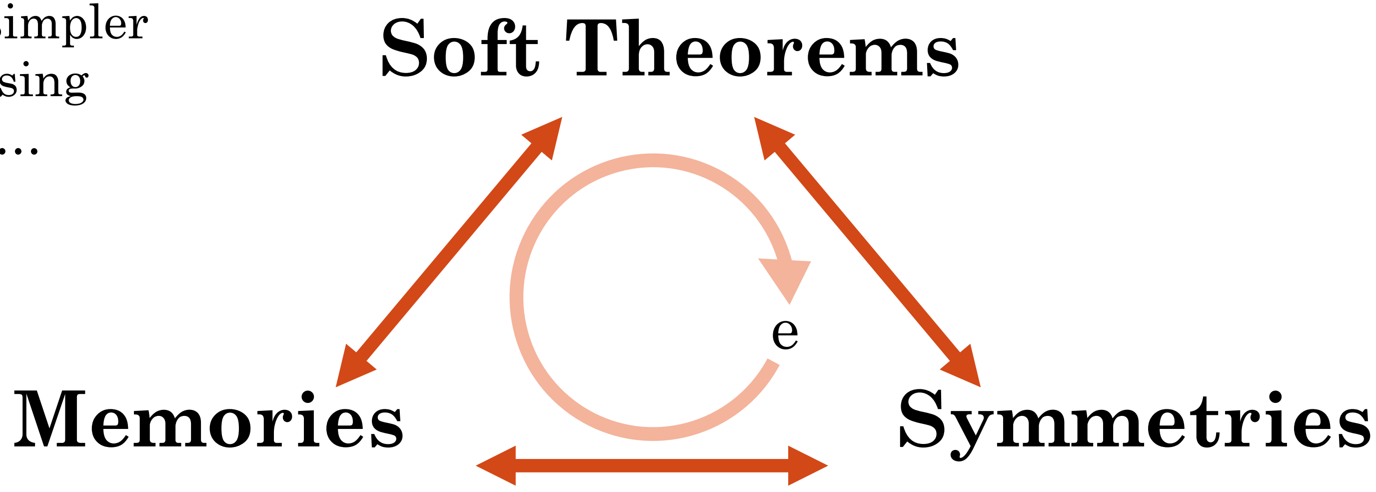

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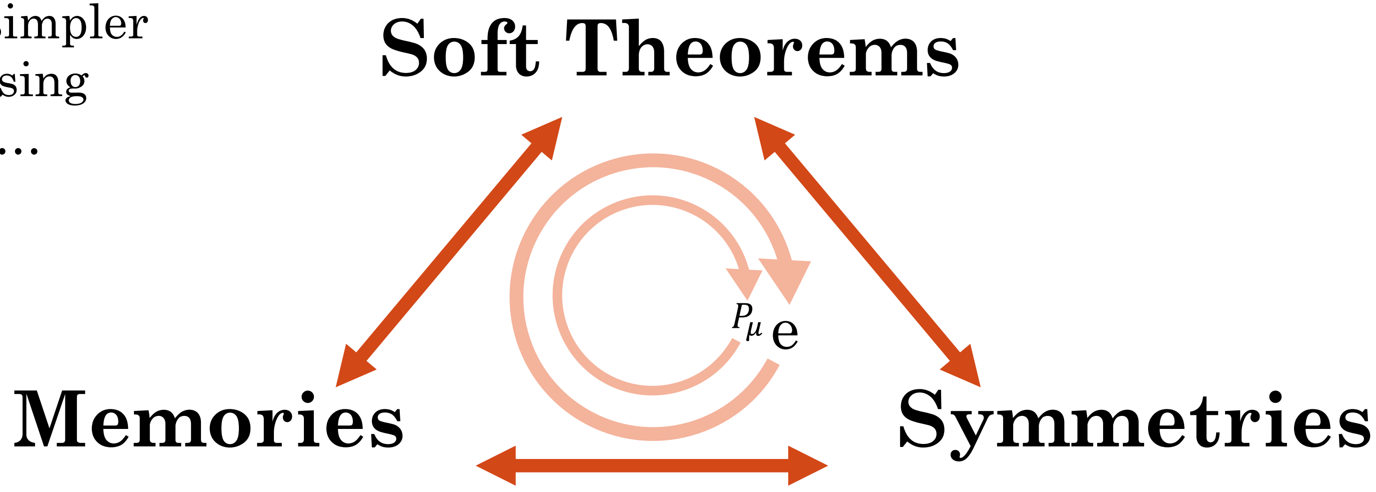

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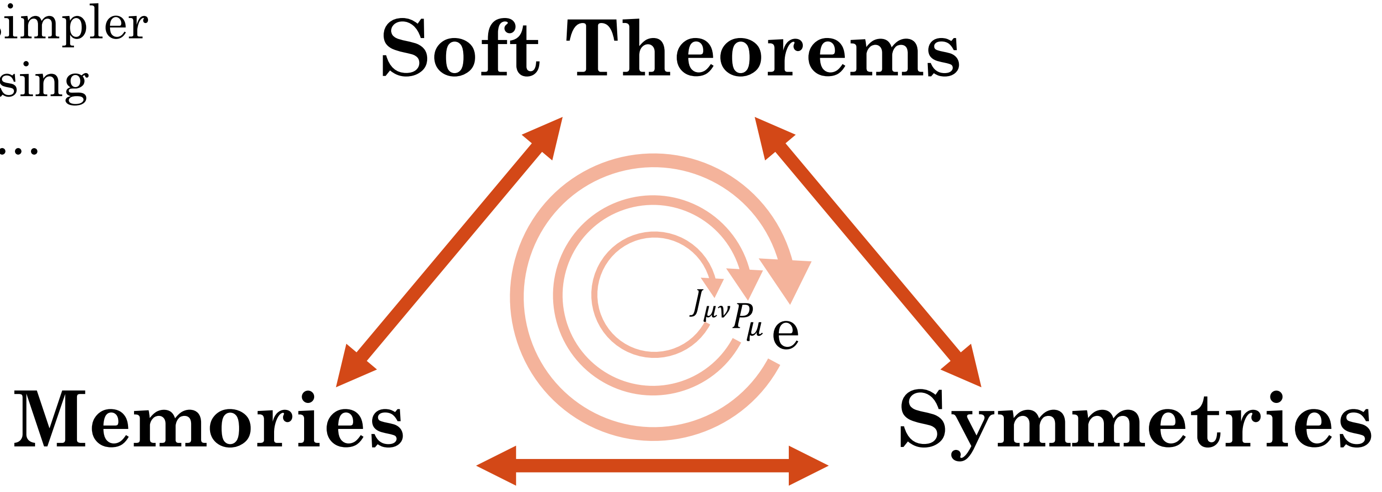

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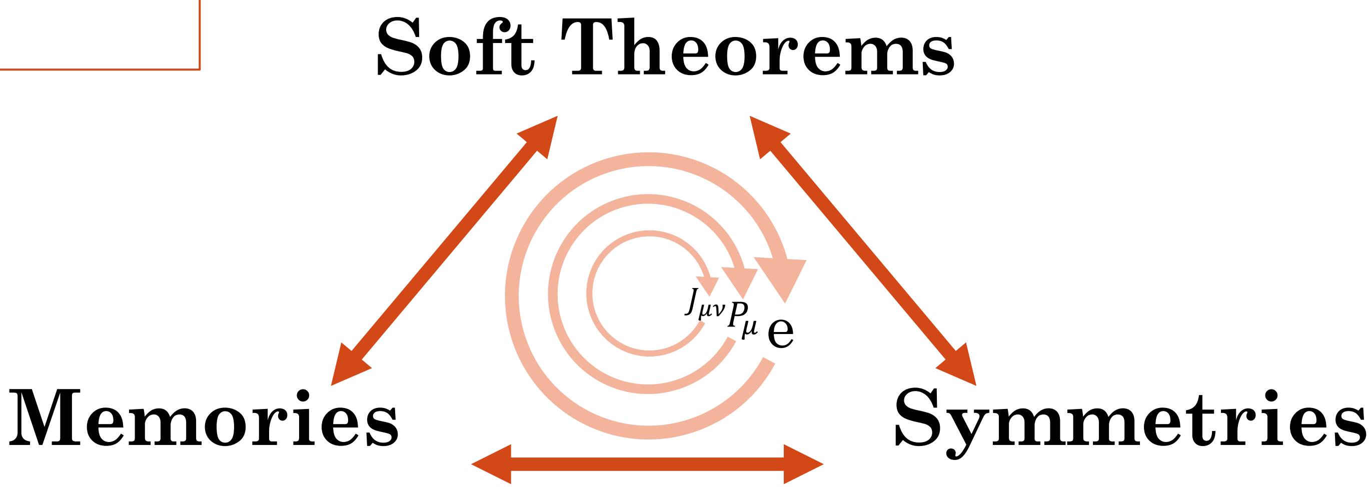
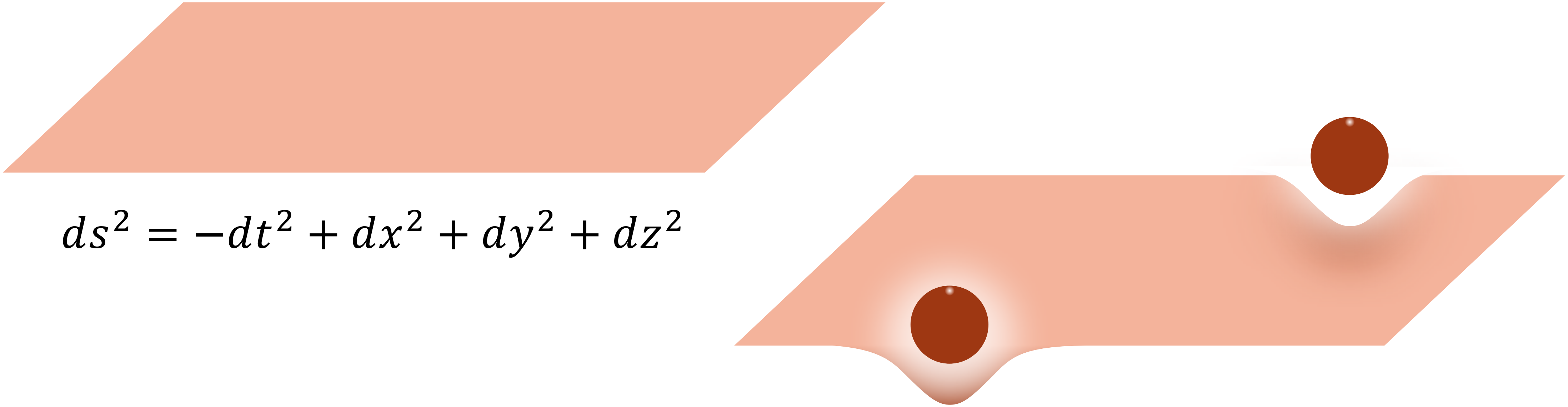
In this manner a new iteration was completed corresponding to ***superrotations***. This generalizes Lorentz transformations and has motivated looking at $\mathcal{S}$-matrix elements in a new basis.



Asymptotically Flat Spacetimes

- We are interested spacetimes with **localized stress tensor sources**.

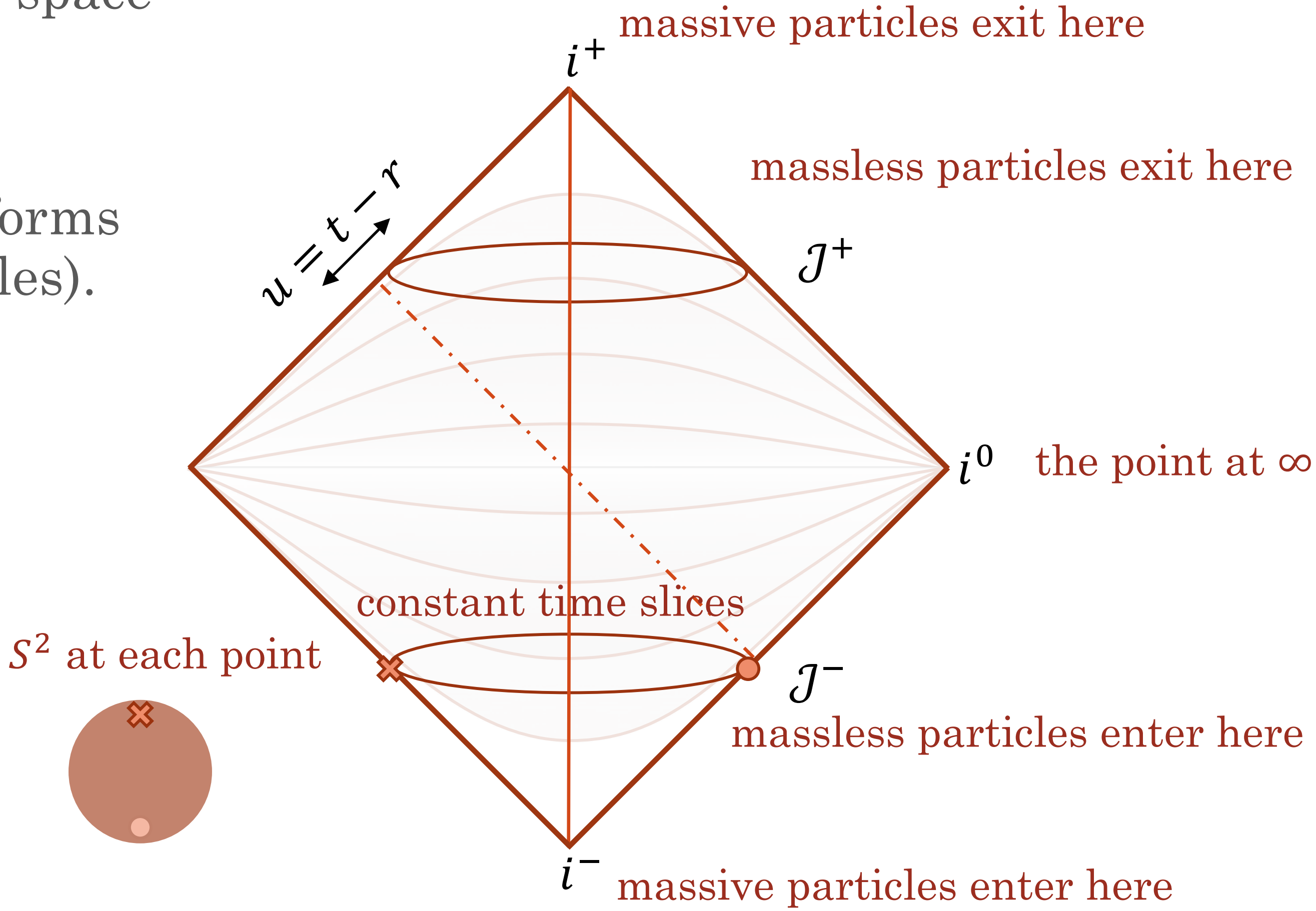


$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

- These can be viewed as deviations from Minkowski space which maintain its asymptotic structure.

Asymptotically Flat Spacetimes

- The boundary of Minkowski space is a null hypersurface
- This is neatly captured by a Penrose diagram (which deforms distances but preserves angles).



Asymptotically Flat Spacetimes

- Following BMS, asymptotically flat spacetimes can be parameterized by an radial expansion of the metric

$$\begin{aligned} ds^2 &= -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} + 2\tfrac{m_B}{r}du^2 \\ &+\big(rC_{zz}dz^2 + D^zC_{zz}dudz + \tfrac{1}{r}(\tfrac{4}{3}N_z - \tfrac{1}{4}\partial_z(C_{zz}C^{zz}))dudz + c.c.\big) + \ldots \end{aligned}$$

- and we can identify the asymptotic symmetry group

$$\mathrm{ASG} = \frac{\text{allowed gauge symmetries}}{\text{trivial gauge symmetries}}$$

- by finding diffeomorphisms which preserve this expansion

$$\begin{aligned} \xi^+ =& (1+\frac{u}{2r})Y^{+z}\partial_z - \frac{u}{2r}D^{\bar{z}}D_zY^{+z}\partial_{\bar{z}} - \frac{1}{2}(u+r)D_zY^{+z}\partial_r + \frac{u}{2}D_zY^{+z}\partial_u + c.c \\ &+ f^+\partial_u - \frac{1}{r}(D^zf^+\partial_z + D^{\bar{z}}f^+\partial_{\bar{z}}) + D^zD_zf^+\partial_r \end{aligned}$$

$$z = e^{i\phi}\tan\frac{\theta}{2} \qquad \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$

$$f^+ = f^+(z,\bar{z}) \qquad \partial_{\bar{z}}Y^{+z} = 0$$

Superrotations

- And breaks into a soft part and a hard part

$$Q^+[Y] = Q^+_S[Y] + Q^+_H[Y]$$

$$Q^+_S[Y] = \frac{1}{2}\int_{\mathcal{I}^+} du d^2 z D_z^3 Y^z u \partial_u C^z_{\bar{z}} \qquad Q^+_H[Y] = \lim_{\Sigma \to \mathcal{I}^+} \int_\Sigma d\Sigma\, \xi^\mu n^\nu_\Sigma T^M_{\mu\nu}$$

- Such that the subleading soft graviton theorem can be used to evaluate Q_S in S-matrix element insertions.

$$\langle out|a_-(q)\mathcal{S}|in\rangle = \left(S^{(0)-} + S^{(1)-}\right)\langle out|\mathcal{S}|in\rangle + \mathcal{O}(\omega)$$

$$S^{(0)-} = \sum_k \frac{(p_k \cdot \epsilon^-)^2}{p_k \cdot q} \qquad S^{(1)-} = -i\sum_k \frac{p_{k\mu}\epsilon^{-\mu\nu}q^\lambda J_{k\lambda\nu}}{p_k \cdot q}$$

Currents

- More generally, soft modes corresponding to other asymptotic symmetries also map to currents.

$$
\begin{tikzcd}
\begin{array}{c}\text{4D} \\ \text{Gravity,} \\ \Lambda=0\end{array} \arrow[r, dotted] & \begin{array}{c}\text{2D} \\ \text{CFT}\end{array} \\
\text{soft operator} \arrow[u, "\begin{array}{c}\text{Ward identity} \\ \text{asym. sym. grp.}\end{array}"] \arrow[r] & \text{current} \arrow[u, "\begin{array}{c}\text{Ward identity} \\ \text{holo. current}\end{array}"']
\end{tikzcd}
$$

Beyond Currents

- Also, one can extend this map beyond the soft limit.

$$\begin{tikzcd}
\begin{matrix}\text{4D} \\ \text{Gravity,} \\ \Lambda=0\end{matrix} \arrow[rr] & & \begin{matrix}\text{2D} \\ \text{CFT}\end{matrix} \\
\begin{matrix}\text{soft} \\ \text{operator}\end{matrix} \arrow[u, "\begin{matrix}\text{Ward identity} \\ \text{asym. sym. grp.}\end{matrix}"] \arrow[rr] & & \text{current} \arrow[u, "\begin{matrix}\text{Ward identity} \\ \text{holo. current}\end{matrix}"']
\end{tikzcd}$$

$4D \rightarrow 2D$ Map

- We can perform this map!

  - Instead of plane waves, our wavefunctions are conformal primaries parameterized by a conformal dimension Δ and direction on the celestial sphere

$$\phi_{\Delta,m}\left(\Lambda^{\mu}{}_{\nu}X^{\nu};\frac{aw+b}{cw+d},\frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}\right)=|cw+d|^{2\Delta}\phi_{\Delta,m}\left(X^{\mu};w,\bar{w}\right)$$

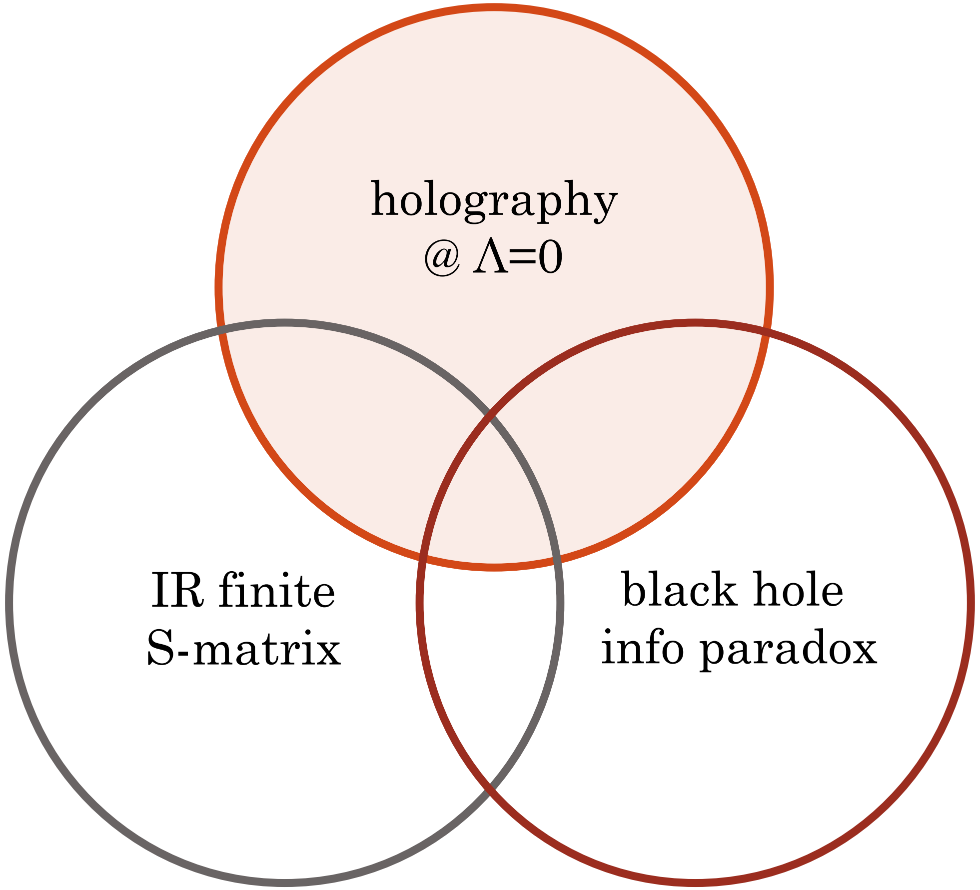
  - In each case the 2D correlator can be computed from a 4D S-matrix element via an integral transform.

$$\tilde{\mathcal{A}}_{\Delta_1,\cdots,\Delta_n}(w_i,\bar{w}_i)\equiv\prod_{k=1}^{n}\int_0^{\infty}d\omega_k\omega_k^{i\lambda_k}\,\mathcal{A}(\omega_k q_k^{\mu}) \qquad m=0$$

$$\widetilde{\mathcal{A}}(\Delta_i,\vec{w}_i)\equiv\prod_{k=1}^{n}\int_{H_{d+1}}[d\hat{p}_k]\,G_{\Delta_k}(\hat{p}_k;\vec{w}_k)\ \mathcal{A}(\pm m_i\hat{p}_i^{\mu}) \qquad m\neq 0$$

Progress

- We have the first steps towards constructing a dictionary from 4D scattering amplitudes to 2D CFT correlators…
- In which certain ‘soft’ IR modes get mapped to 2D currents…
- And imply an infinite number of conserved charges.



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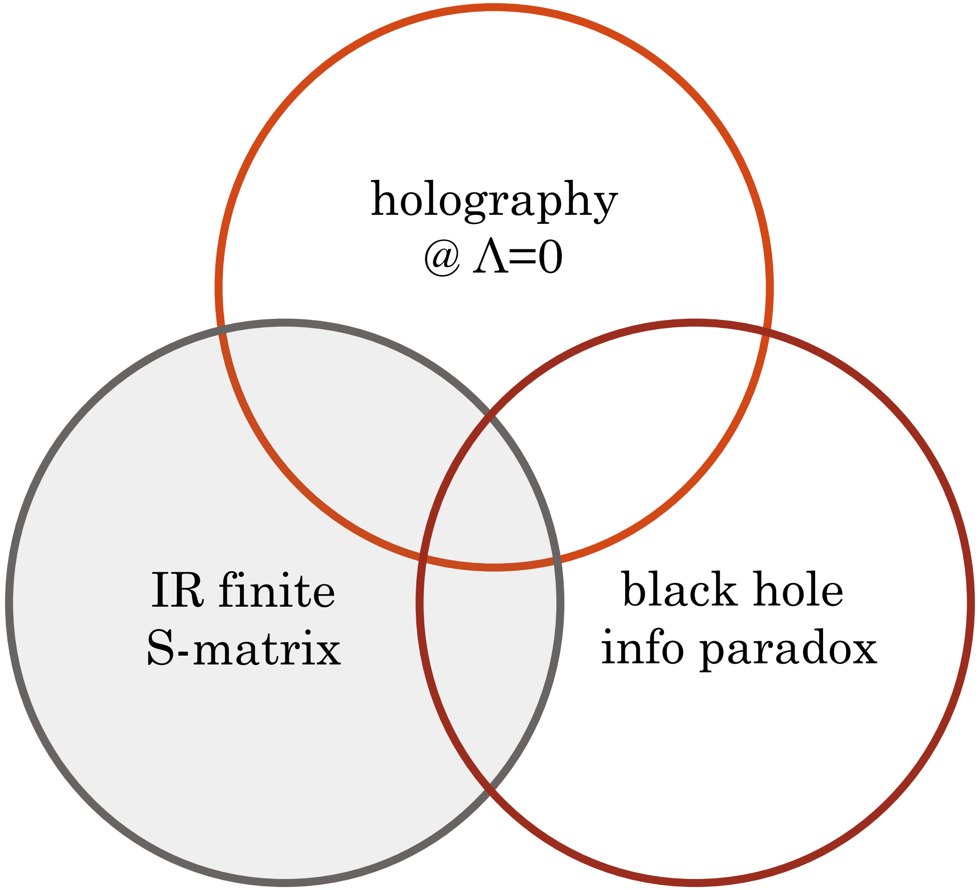

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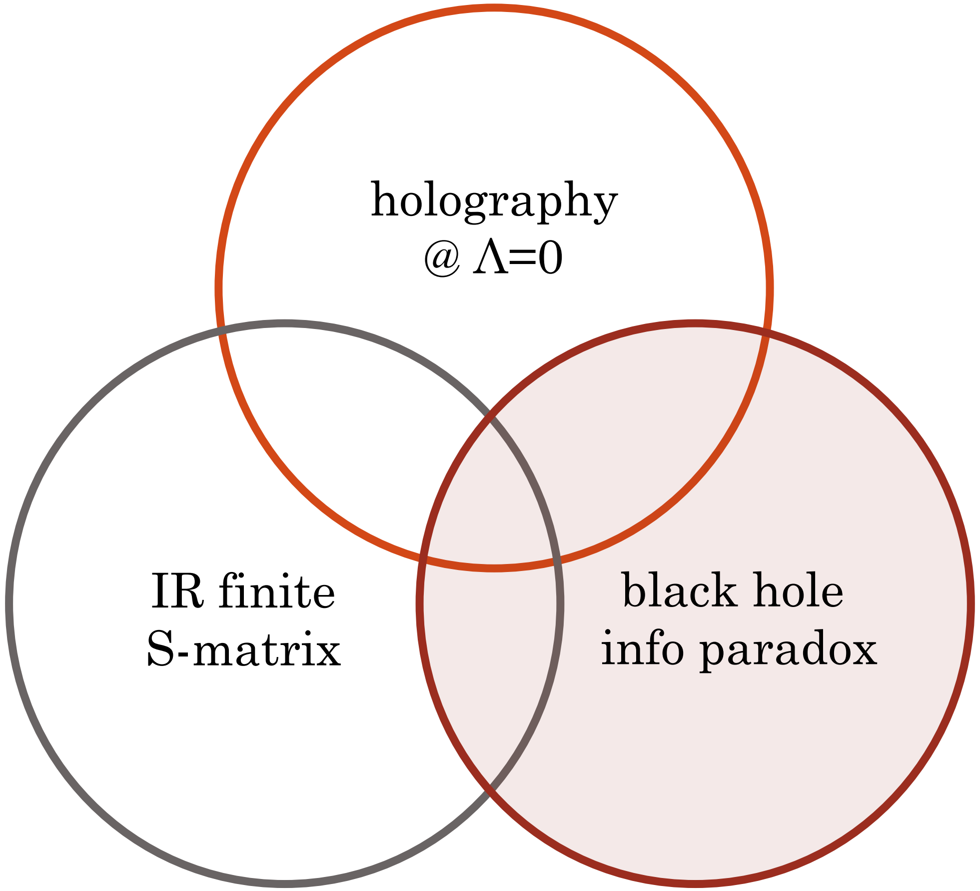

How far have we come?

- We have an interesting interpretation of IR divergences as harbingers of symmetry enhancement.
- These infinite number of conserved charges has the potential to add ‘soft hair’ to black holes.
- We are still a ways away from understanding the putative 2D dual CFT, but have made progress on the kinematics/symmetry front.