Memories, Symmetries, & Soft Theorems

SABRINA GONZALEZ PASTERSKI
Scattering in Quantum Gravity

\[ G_{\mu\nu} = 8\pi G T_{\mu\nu} \Rightarrow S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \ R \Rightarrow g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \Rightarrow \text{Non-renormalizeable} \]

- Significant quantum effects expected at short distances, Planck scale.
  \( \Rightarrow \) Look for UV-complete theory... Strings?

- Strominger et al: Is there something we can learn from long distance, IR effects?
  \( \Rightarrow \) Use symmetries to constrain scattering.
\( S \)-matrix Constraints from Symmetries

• Noether’s Theorem: Continuous Symmetries ⇒ Conservation Laws

• More Symmetries ⇒ More Constraints on \( S \)-matrix

• Modus Operandi:
  ➢ Look for larger set of “physical” symmetries
  ➢ Motivate via properties of low energy scattering

Soft Theorems

Memories

Symmetries
Outline

• Define Scattering Arena
  
  Asymptotically Flat Spacetimes, Null Infinity

• Consider Solution Properties
  
  Boundary Behavior, Memory Effects

• Connect Position and Momentum Space
  
  Soft Theorems, Ward Identities
Outline

• Define Scattering Arena
  *Asymptotically Flat Spacetimes, Null Infinity*

• Consider Solution Properties
  *Boundary Behavior, Memory Effects*

• Connect Position and Momentum Space
  *Soft Theorems, Ward Identities*

---

Soft Theorems

Memories

Symmetries
Asymptotically Flat Spacetimes

\[ ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \]

Want to consider non-trivial gravitational backgrounds that are ``close” to being flat

- Approach flat spacetime far away from sources
Asymptotically Flat Spacetimes

To do so, need to formalize our notion of the boundary of flat Minkowski space

This structure will carry over to asymptotically flat spacetimes
Asymptotically Flat Spacetimes

Distances are rescaled but the causal structure is preserved.

Massive particles enter here.
Massless particles enter here.
Massless particles exit here.
Massive particles exit here.

The point at infinity is denoted by $i^0$. The $i^+$ and $i^-$ are future and past horizons, respectively.
Asymptotically Flat Spacetimes

\( S^2 \) at each point

constant time slices

massive particles exit here

massless particles exit here

massless particles enter here

massive particles enter here

the point at \( \infty \)
Outline

✓ Define Scattering Arena
   Asymptotically Flat Spacetimes, Null Infinity

• Consider Solution Properties
   Boundary Behavior, Memory Effects

• Connect Position and Momentum Space
   Soft Theorems, Ward Identities

Soft Theorems

Memories

Symmetries
Determine what data is needed to specify a classical solution

- eg. \((x, p)@t_0\) for a point particle

In gauge theories there are constraints that need to be satisfied on each slice.
Solution Behavior & Free Data

Ashtekar 1980’s

constant time slices

$|\Psi\rangle_{in}$

$|\Psi\rangle_{out}$
Solution Behavior & Free Data

BMS studied boundary behavior of metric at null infinity, isolated the free data, and studied residual gauge symmetries that act non-trivially on this data:

- \( g_{\mu\nu} = g_{\mu\nu}^{(0)} + \frac{1}{r} g_{\mu\nu}^{(1)} + \ldots \)
- \( g_{\mu\nu} \rightarrow g_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu \)

A certain class of \( \xi \) preserve the falloff form.
Solution Behavior & Free Data

BMS studied boundary behavior of metric at null infinity, isolated the free data, and studied residual gauge symmetries that act non-trivially on this data

- $g_{\mu\nu} = g^{(0)}_{\mu\nu} + \frac{1}{r} g_{\mu\nu} + ...$
- $g_{\mu\nu} \to g_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$

A certain class of $\xi$ preserve the falloff form

A similar story occurs in E&M

- $A_\mu = A^{(0)}_\mu + \frac{1}{r} A^{(1)}_\mu + ...$
- $A_\mu \to A_\mu + \nabla_\mu \lambda$

$E_{rad} \propto \frac{1}{r}$

$E_{coul} \propto \frac{1}{r^2}$
Don’t want boundary conditions to be too restrictive so as to disallow typical scattering processes

*Memory effects* are radiation observables, whose values are non-zero in typical scattering processes, and which are related to the sourcing scatterers via constraint equations
Don’t want boundary conditions to be too restrictive so as to disallow typical scattering processes

*Memory effects* are radiation observables, whose values are non-zero in typical scattering processes, and which are related to the sourcing scatterers via constraint equations
Don’t want boundary conditions to be too restrictive so as to disallow typical scattering processes.

*Memory effects* are radiation observables, whose values are non-zero in typical scattering processes, and which are related to the sourcing scatterers via constraint equations.
Outline

- Define Scattering Arena
  Asymptotically Flat Spacetimes, Null Infinity

- Consider Solution Properties
  Boundary Behavior, Memory Effects

- Connect Position and Momentum Space
  Soft Theorems, Ward Identities

Soft Theorems

Memories

Symmetries
Ward Identities from Soft Theorems

$S$-matrix elements are typically computed between in and out states of particles with definite momentum $p^\mu$

Soft theorems describe a relation between a given scattering process and one with an extra gauge particle as its energy approaches zero
Ward Identities from Soft Theorems

\[ \langle \text{out} ; \omega \hat{q}, \pm | S | \text{in} \rangle = (S^{(0)} + \omega S^{(1)}) \langle \text{out} | S | \text{in} \rangle + \ldots \]

Soft theorems describe a relation between a given scattering process and one with an extra gauge particle as its energy approaches zero.
Ward Identities from Soft Theorems

\[ \langle \text{out}; \omega \hat{q}, \pm |\text{in}\rangle = (S^{(0)} + \omega S^{(1)}) \langle \text{out}|\text{in}\rangle + \cdots \]

we can connect
\[ \hat{q}_{\text{pos}} \Leftrightarrow \hat{q}_{\text{mom}} \]
and
\[ \lim_{\omega \to 0} \Leftrightarrow \int du \]
Ward Identities from Soft Theorems

• Strominger [arXiv:1308.0589 arXiv:1312.2229]:
  - Soft theorems can be used to construct Ward identities
  - Large gauge transformations $S^2$ dependent rather than constant

✓ Modus Operandi:
  - Look for larger set of ``physical'' symmetries
  - Motivate via properties of low energy scattering
A Triangle of Relations

Soft Theorems

Memories ←→ Symmetries
A Triangle of Relations

Soft Theorems

Memories ↔ Symmetries

what we’re after:
More Symmetries ⇒ More Constraints on $\mathcal{S}$-matrix
A Triangle of Relations

Soft Theorems

Memories ↔ Symmetries

what we’re after:
More Symmetries \Rightarrow More Constraints on S\text{-matrix}

non-zero net effects in a typical scattering process forces us to have asymptotic behavior that allows them, these extra symmetries then act non-trivially
A Triangle of Relations

relate $S$-matrix elements for states with and without extra soft gauge particle

**Soft Theorems**

**Memories** ↔ **Symmetries**

what we’re after:
More Symmetries $\Rightarrow$ More Constraints on $S$-matrix

non-zero net effects in a typical scattering process forces us to have asymptotic behavior that allows them, these extra symmetries then act non-trivially
A Triangle of Relations

relate $S$-matrix elements for states with and without extra soft gauge particle

**Soft Theorems**

*Fourier transform: long time $\leftrightarrow$ low energy*


**Memories**

*Step (net change) vs baseline (starting point)*

**Symmetries**

*What we’re after: More Symmetries $\Rightarrow$ More Constraints on $S$-matrix*

non-zero net effects in a typical scattering process forces us to have asymptotic behavior that allows them, these extra symmetries then act non-trivially.
A Triangle of Relations

i) Weinberg – photon $O\left(\frac{1}{\omega}\right)$
ii) Weinberg – graviton $O\left(\frac{1}{\omega}\right)$
iii) Cachazo & Strominger – graviton $O(1)$

Soft Theorems

Memories
i) Liénard-Wiechert / Bieri & Garfinkle
ii) Zeldovich & Polnarev / Christodoulou
iii) Pasterski, Strominger, & Zhiboedov

Symmetries
i) e-charge
ii) $P_\mu$
iii) $J_{\mu\nu}$

(global)  (asymptotic)

i) large U(1)
ii) supertranslations
iii) superrotations
A Triangle of Relations

\begin{itemize}
\item[i)] Weinberg – photon \(O(\frac{1}{\omega})\)
\item[ii)] Weinberg – graviton \(O(\frac{1}{\omega})\)
\item[iii)] Cachazo & Strominger – graviton \(O(1)\)
\end{itemize}

**Soft Theorems**

\begin{itemize}
\item[i)] Liénard-Wiechert / Bieri & Garfinkle
\item[ii)] Zeldovich & Polnarev / Christodoulou
\item[iii)] Pasterski, Strominger, & Zhiboedov
\end{itemize}

**Memories**

**Symmetries**

\begin{itemize}
\item[i)] e-charge
\item[ii)] \(P_\mu\)
\item[iii)] \(J_{\mu\nu}\)
\end{itemize}

\begin{itemize}
\item[i)] (global) \(e\)-charge
\item[ii)] (asymptotic) \(P_\mu\)
\item[iii)] (asymptotic) \(J_{\mu\nu}\)
\end{itemize}

\(e\)-charge

large \(U(1)\)

supertranslations

superrotations
Conformal Symmetries of the Celestial $S^2$

This triangle of relations guided the completion of a new iteration which has the structure of a Virasoro algebra on the celestial sphere

- Reparameterization invariance in Soft-Collinear Effective Theory
- Highest-weight scattering