Memories, Symmetries, & Soft Theorems

SABRINA GONZALEZ PASTERSKI

Scattering in Quantum Gravity

$G_{\mu\nu} = 8\pi G T_{\mu\nu} \;\Rightarrow\; S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R \;\Rightarrow\; g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \Rightarrow$ Non-renormalizeable

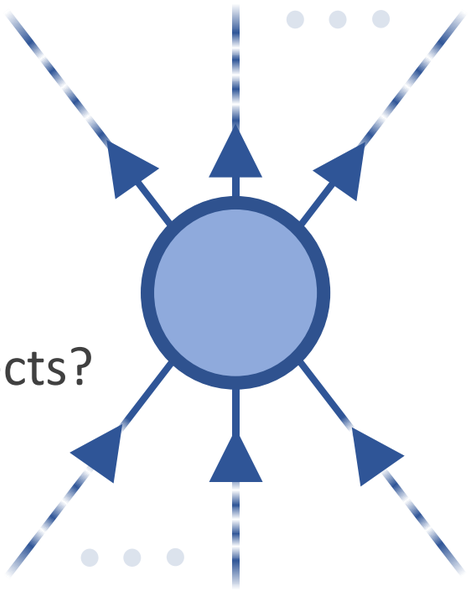
action formulation linearization QFT interpretation

- Significant quantum effects expected at short distances, Planck scale.

⇒ Look for UV-complete theory… Strings?

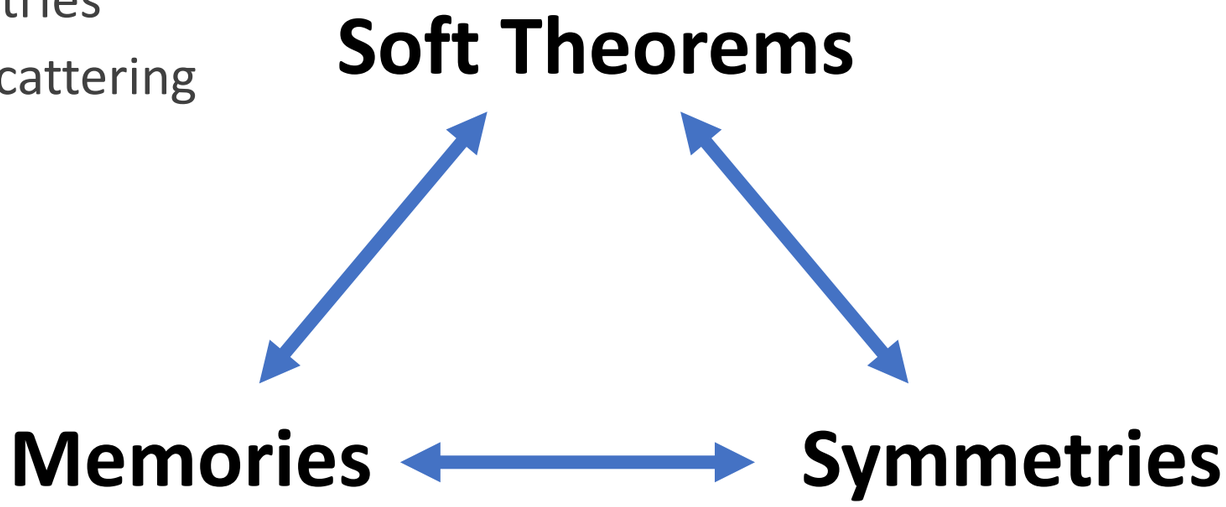
- Strominger et al: Is there something we can learn from long distance, IR effects?

⇒ Use symmetries to constrain scattering.



$\mathcal{S}$-matrix Constraints from Symmetries

- Noether's Theorem: Continuous Symmetries ⇒ Conservation Laws
- More Symmetries ⇒ More Constraints on $\mathcal{S}$-matrix
- Modus Operandi:
  - Look for larger set of ``physical" symmetries
  - Motivate via properties of low energy scattering



Outline

- Define Scattering Arena

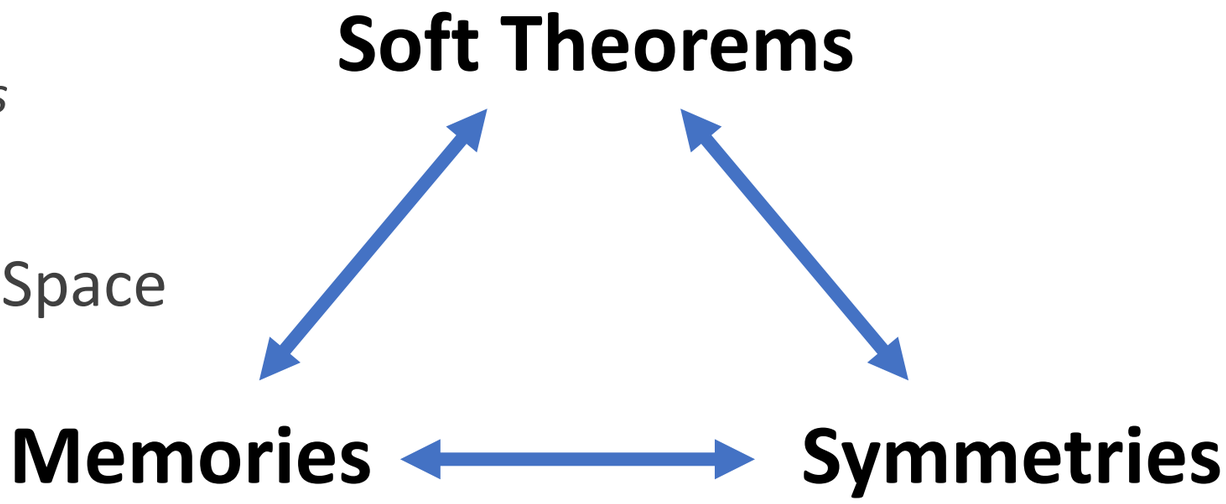
  Asymptotically Flat Spacetimes, Null Infinity

- Consider Solution Properties

  Boundary Behavior, Memory Effects

- Connect Position and Momentum Space

  Soft Theorems, Ward Identities



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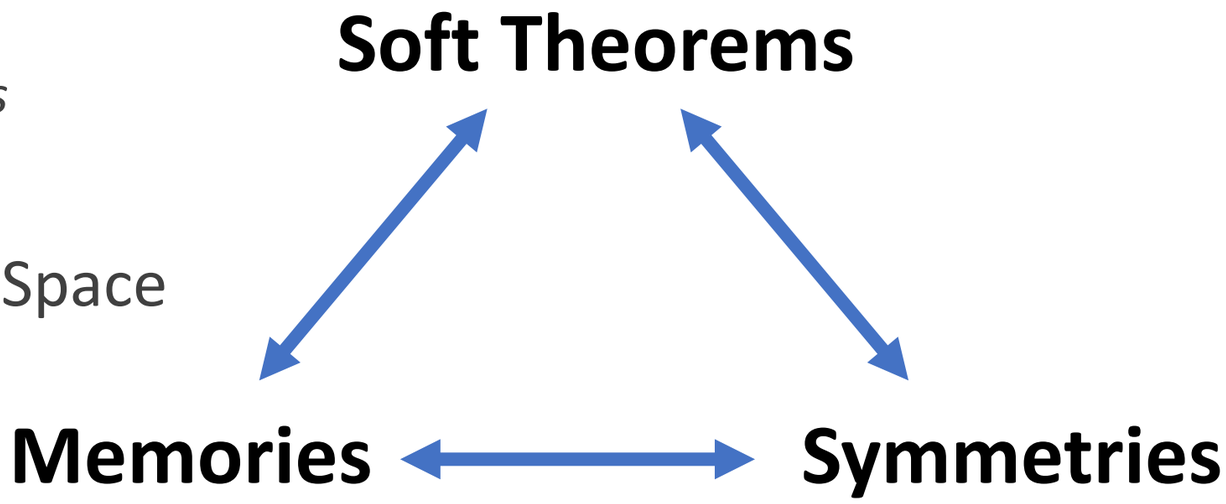


Asymptotically Flat Spacetimes

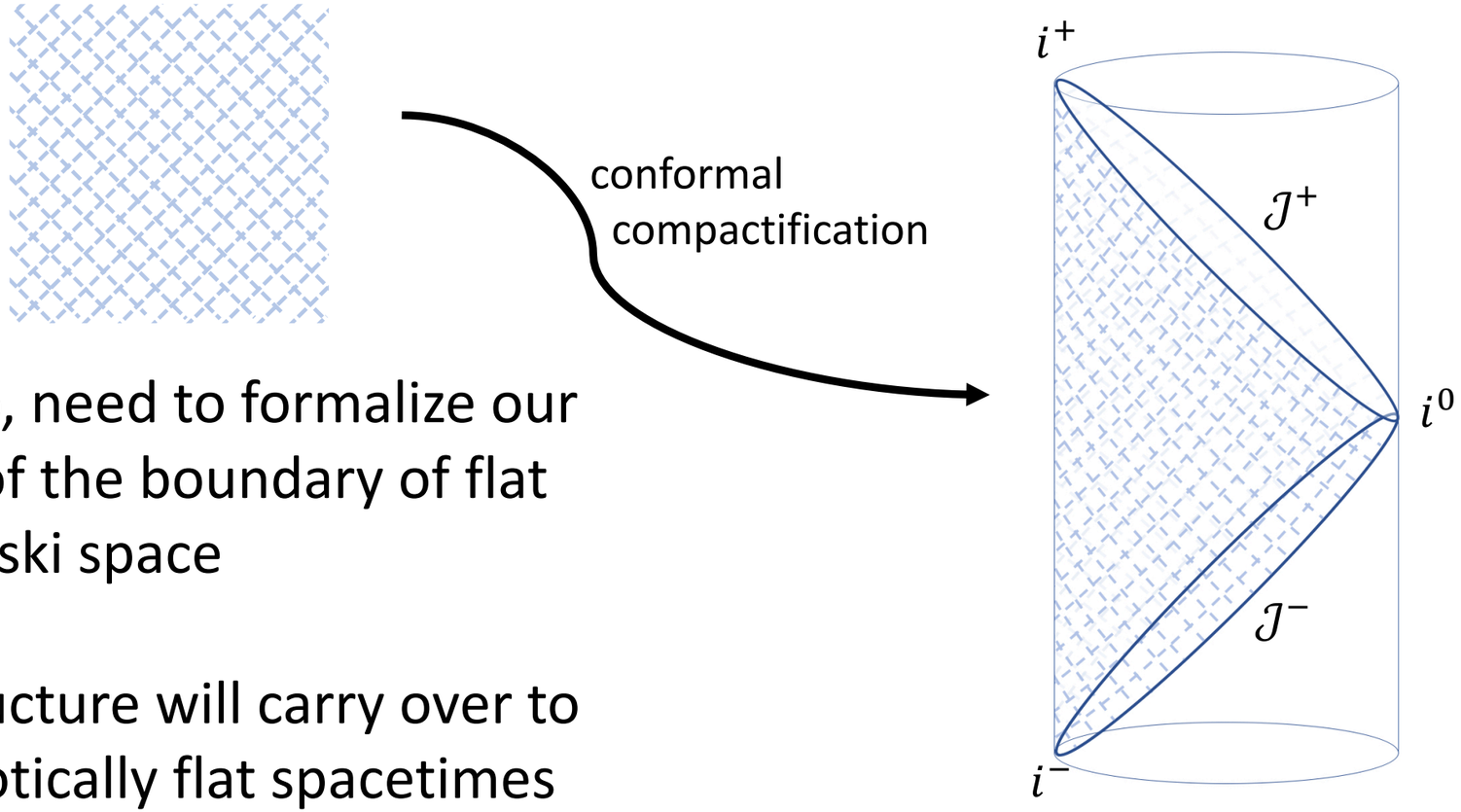
$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

Want to consider non-trivial gravitational backgrounds that are ``close" to being flat

BMS 1960's

- Approach flat spacetime far away from sources

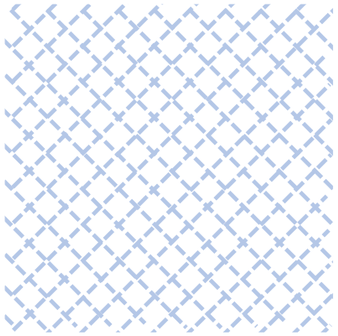
Asymptotically Flat Spacetimes



To do so, need to formalize our notion of the boundary of flat Minkowski space

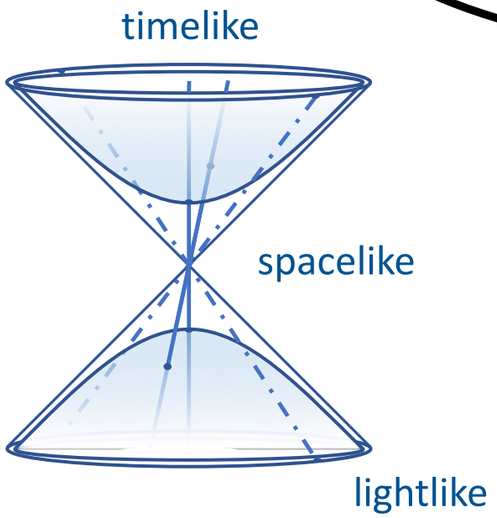
This structure will carry over to asymptotically flat spacetimes

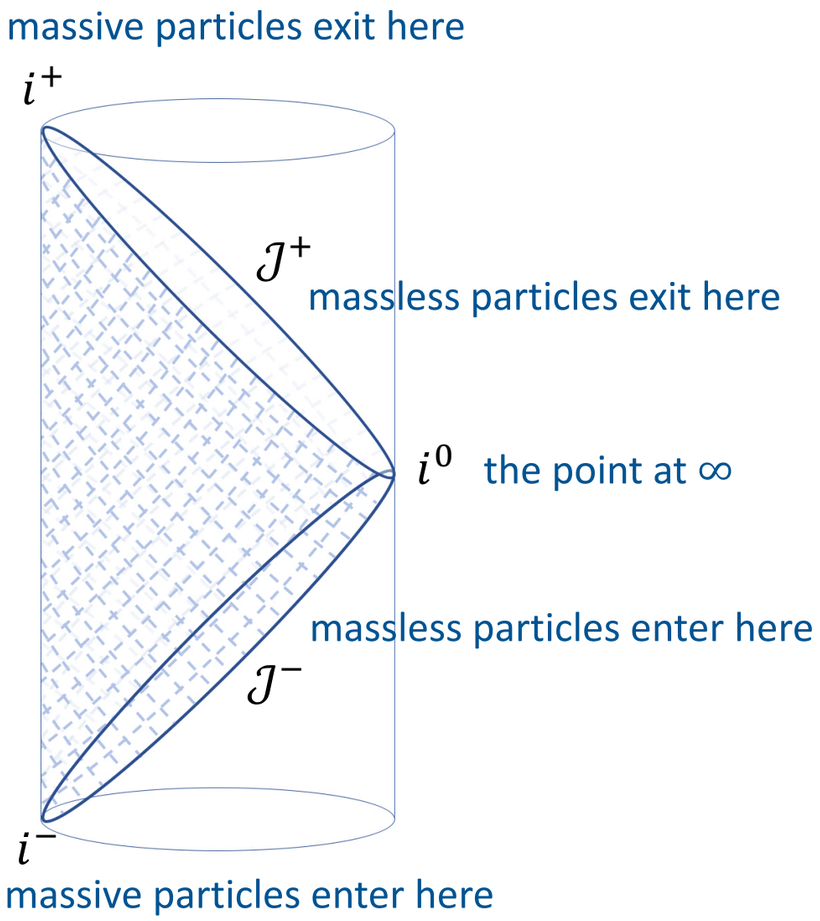
Asymptotically Flat Spacetimes



conformal compactification

Distances are rescaled but the causal structure is preserved





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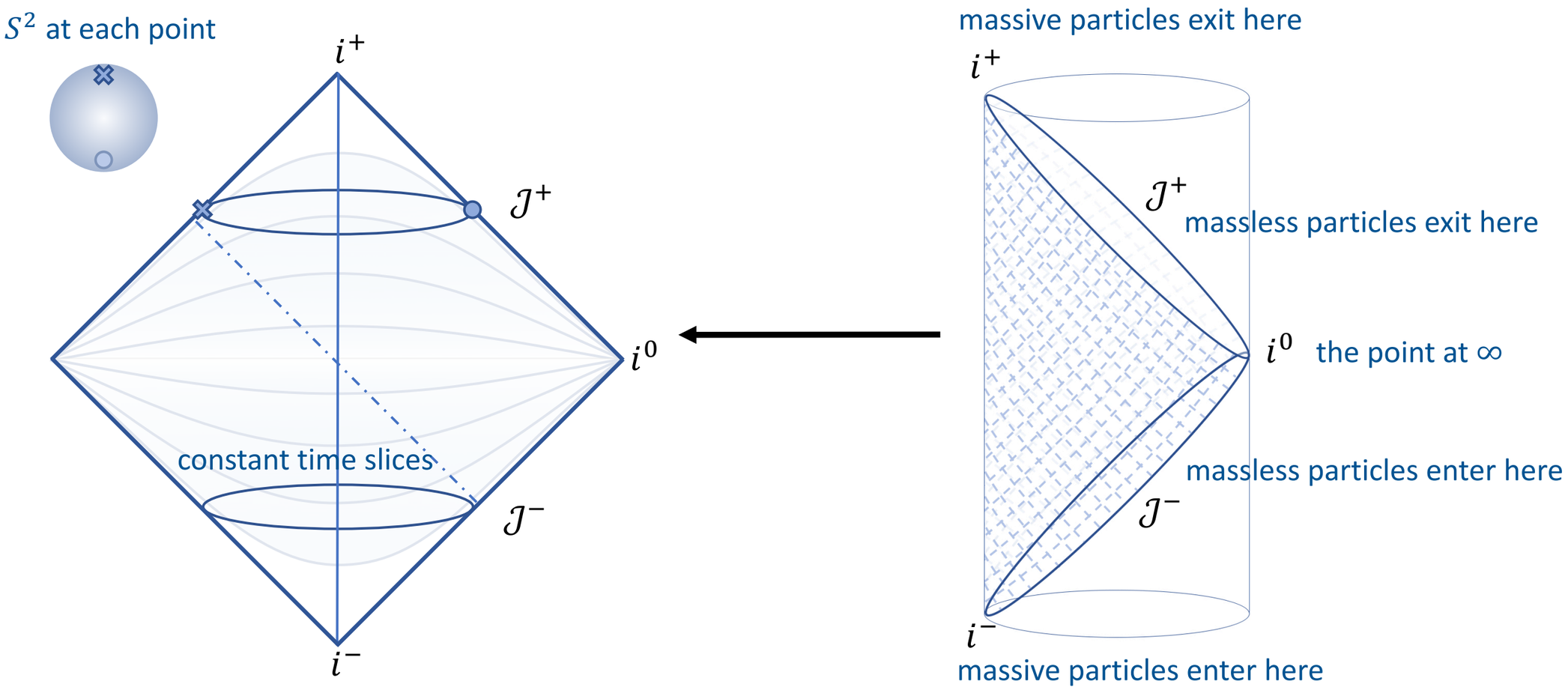

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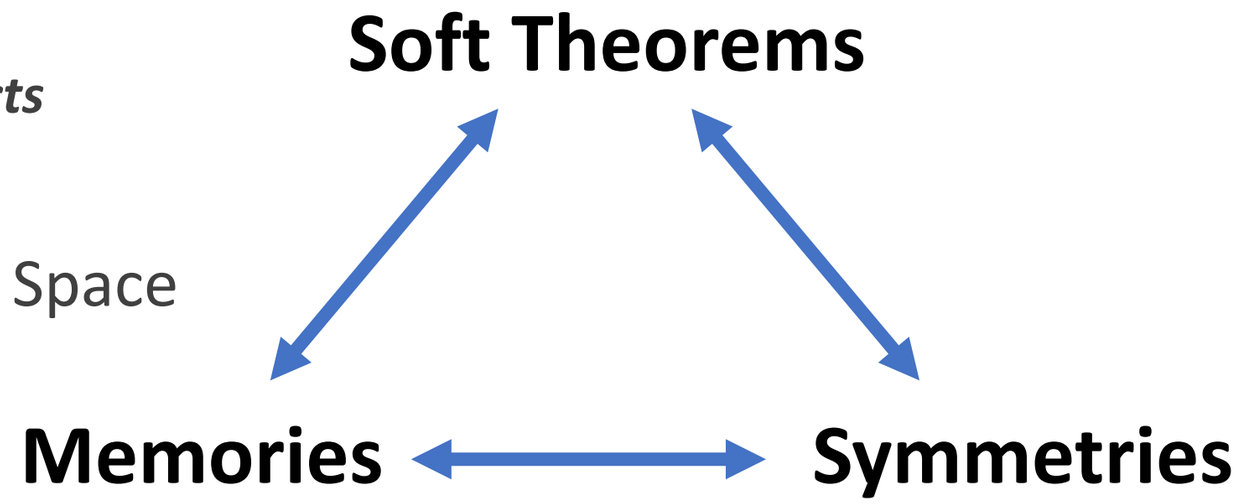
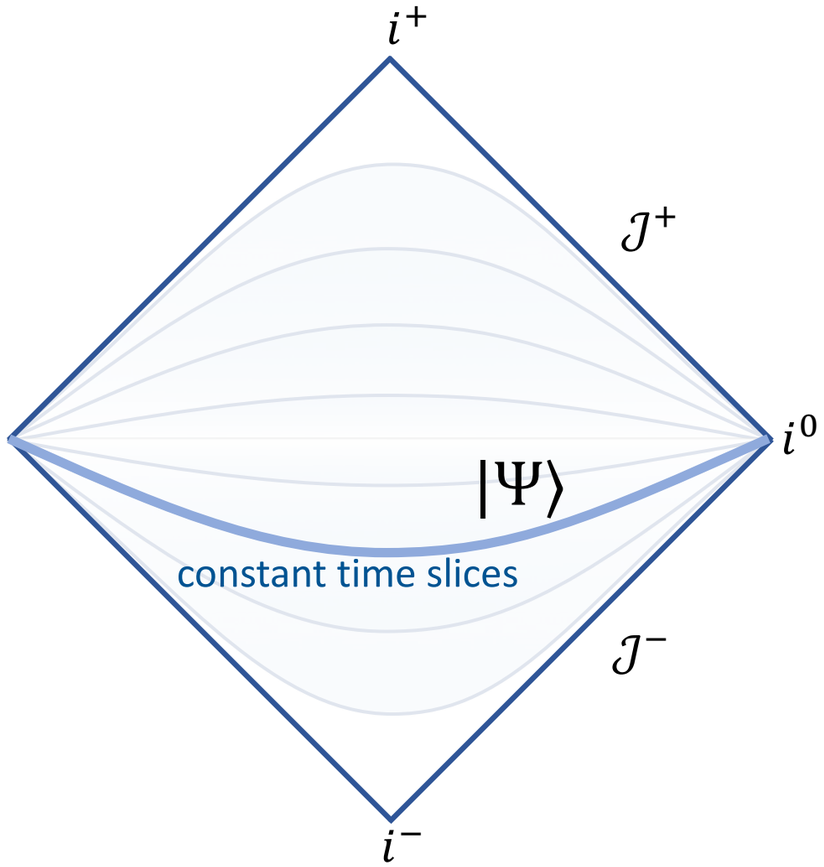
Soft Theorems, Ward Identities



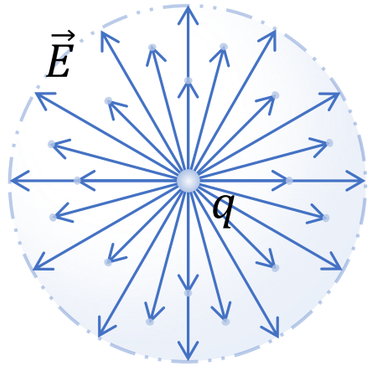
Solution Behavior & Free Data



Determine what data is needed to specify a classical solution

- eg. (x, p) @ t_0 for a point particle

In gauge theories there are constraints that need to be satisfied on each slice



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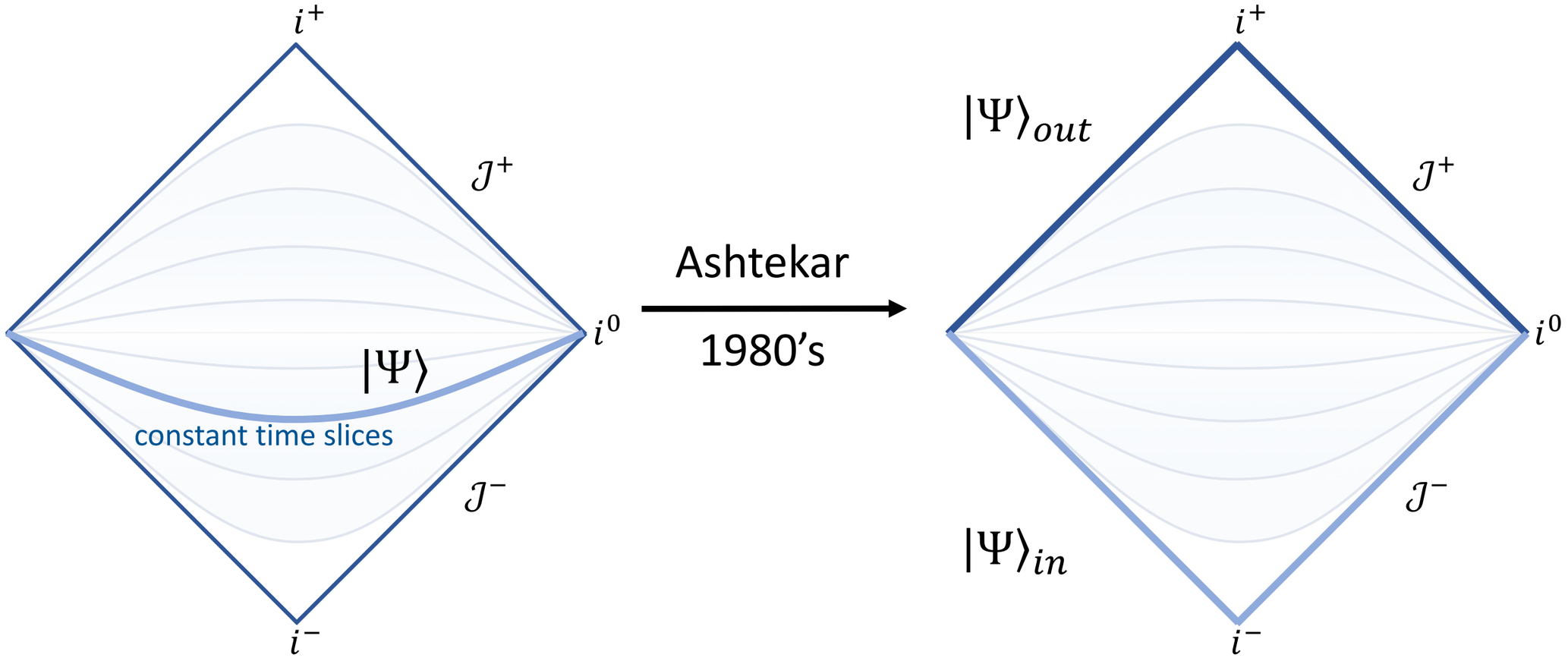
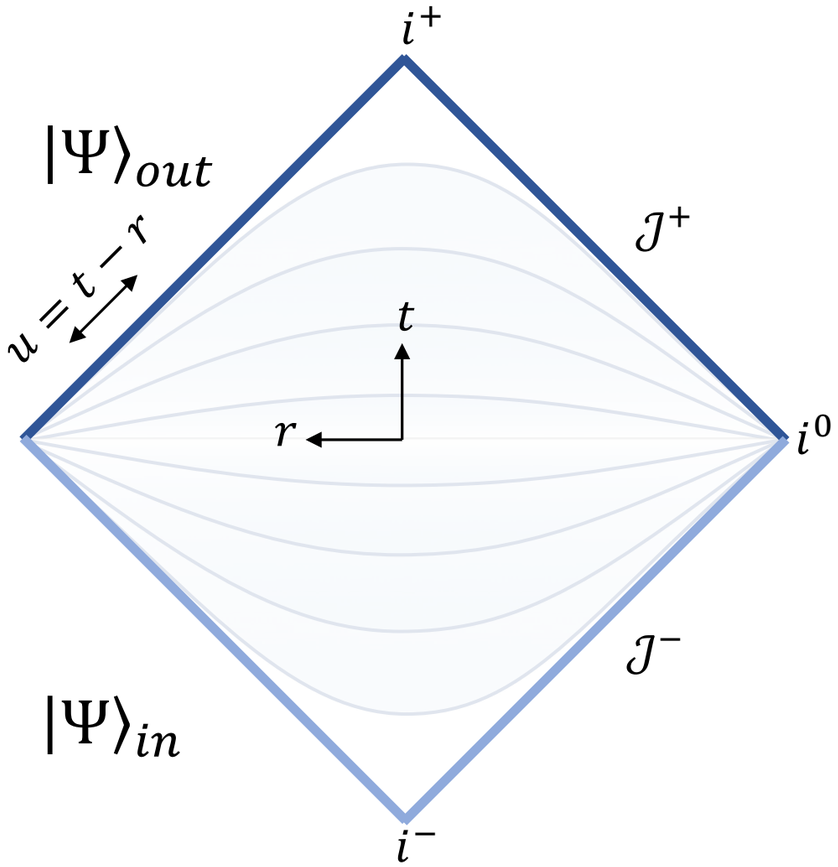


Solution Behavior & Free Data

BMS studied boundary behavior of metric at null infinity, isolated the free data, and studied residual gauge symmetries that act non-trivially on this data

- $g_{\mu\nu} = g_{\mu\nu}^{(0)} + \frac{1}{r} g_{\mu\nu}^{(1)} + \ldots$
- $g_{\mu\nu} \to g_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$

A certain class of ξ preserve the falloff form



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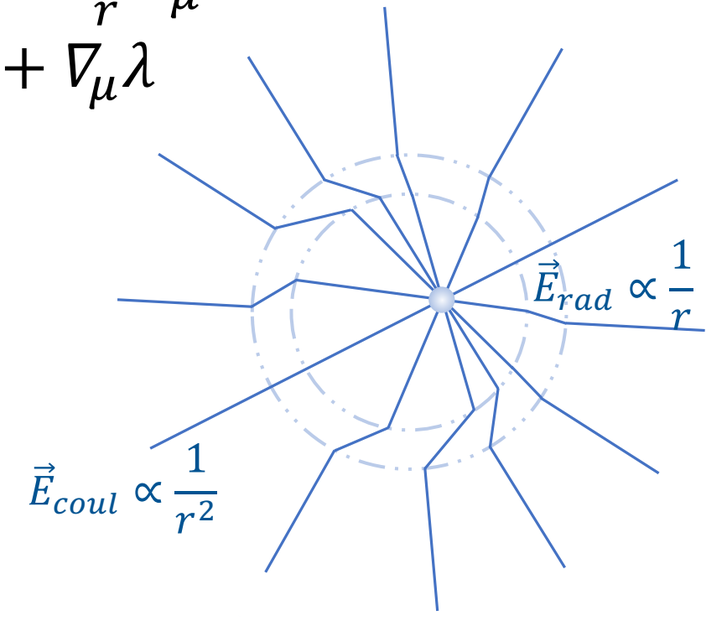
A certain class of ξ preserve the falloff form

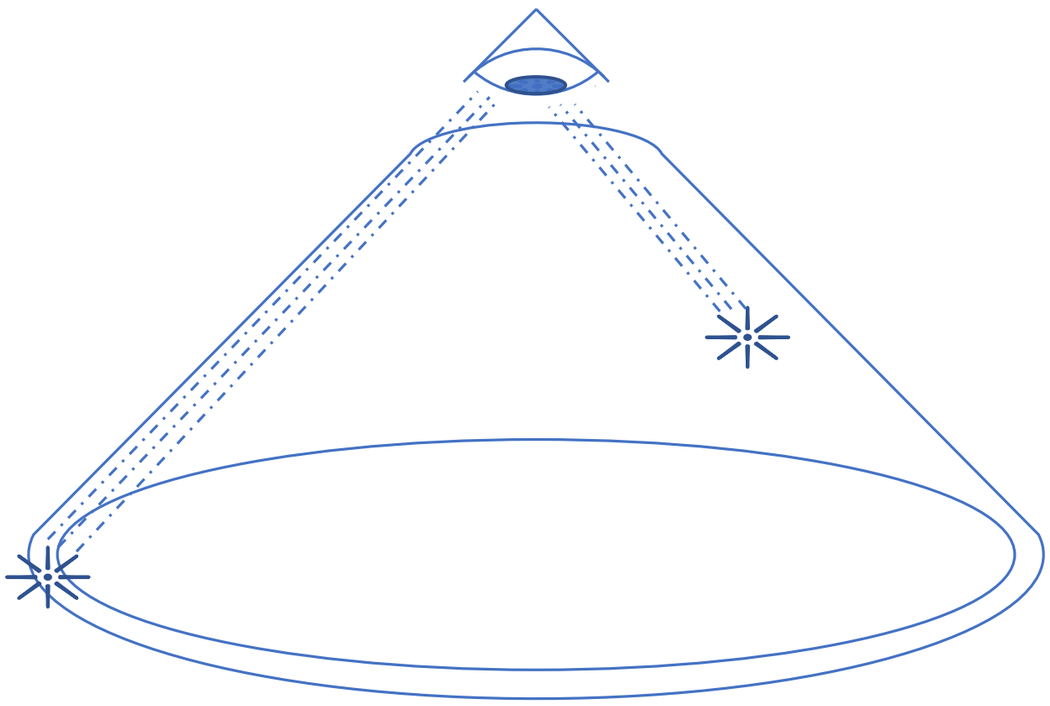
A similar story occurs in E&M

- $A_\mu = A_\mu^{(0)} + \frac{1}{r} A_\mu^{(1)} + \ldots$
- $A_\mu \to A_\mu + \nabla_\mu \lambda$



Solution Behavior & Free Data

Don’t want boundary conditions to be too restrictive so as to disallow typical scattering processes



Memory effects are radiation observables, whose values are non-zero in typical scattering processes, and which are related to the sourcing scatterers via constraint equations

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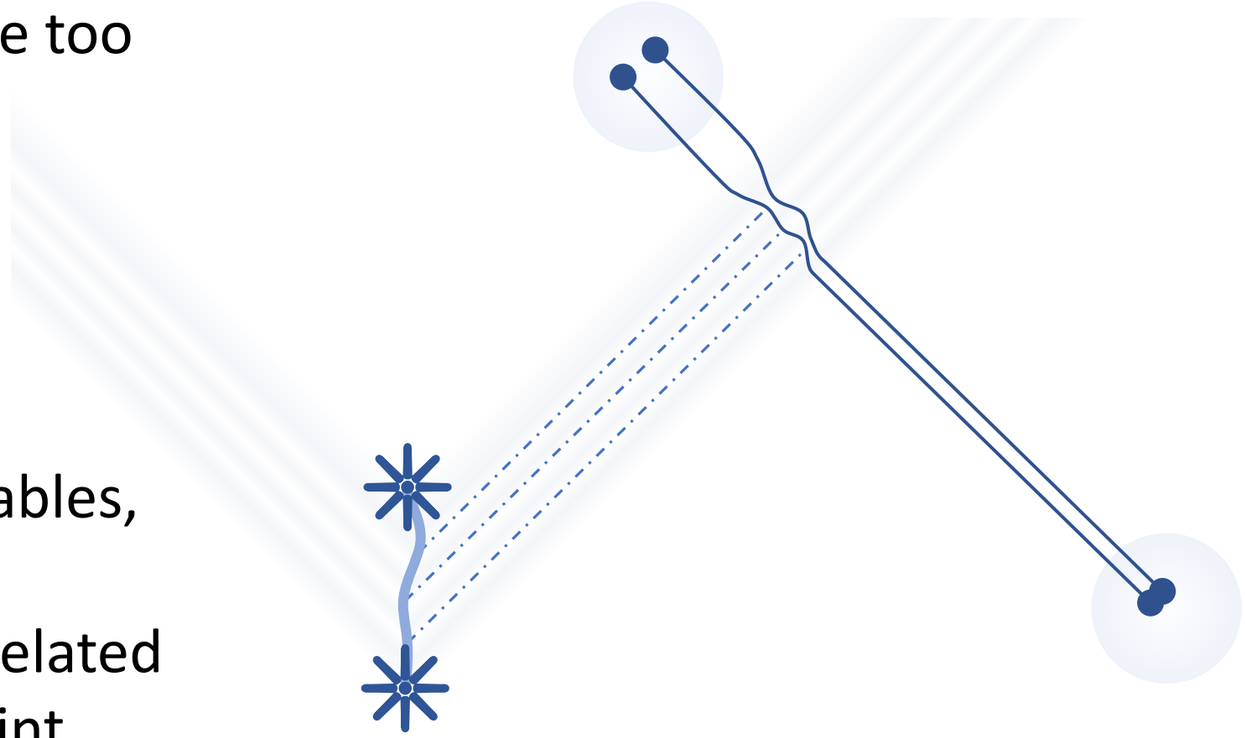

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net displacement of masses or time integrated $\vec{E}_{rad}$

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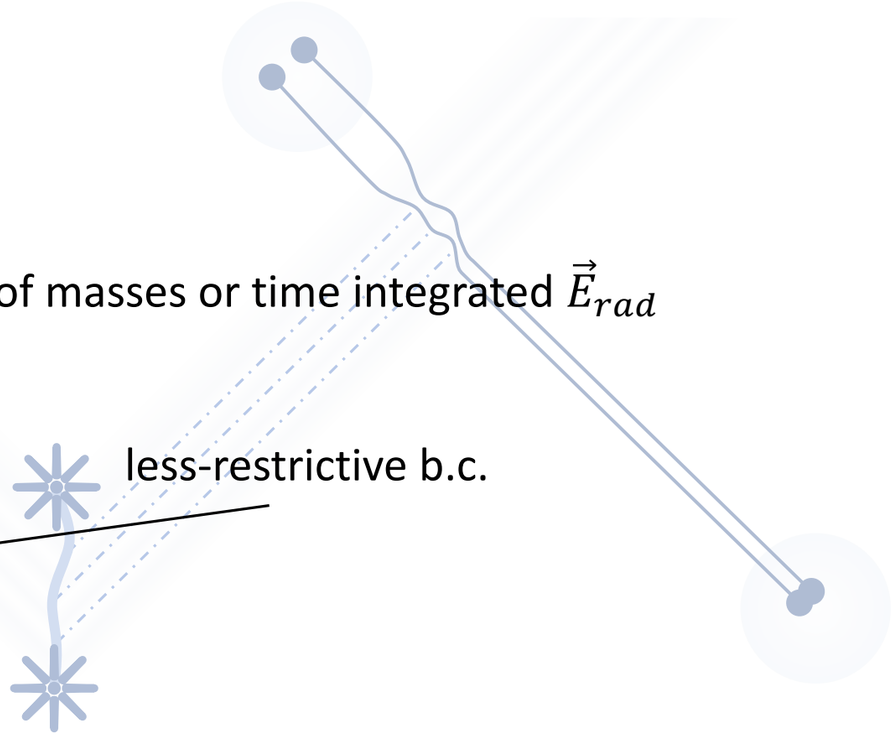

less-restrictive b.c.

connection to soft theorems

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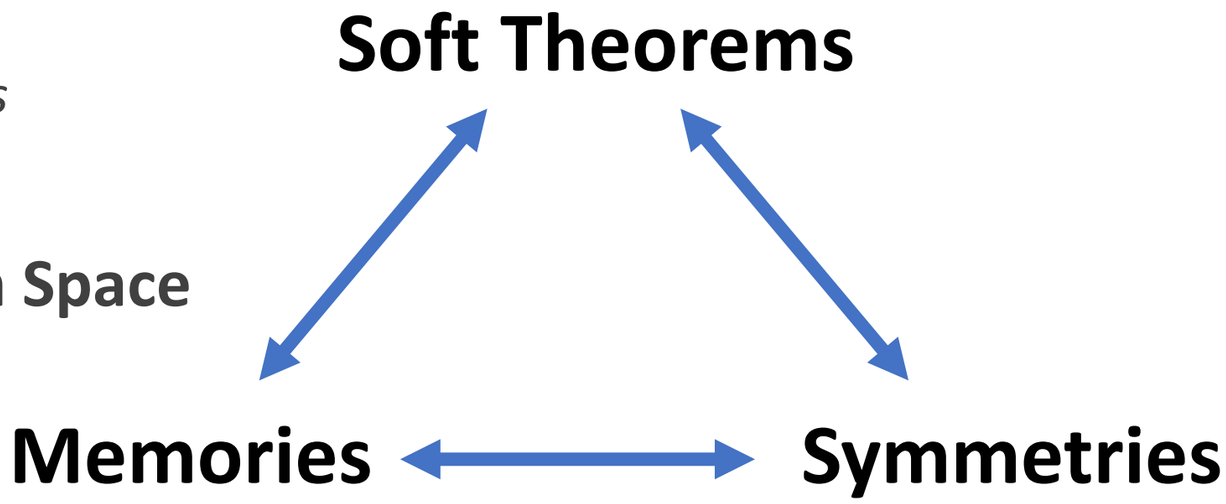
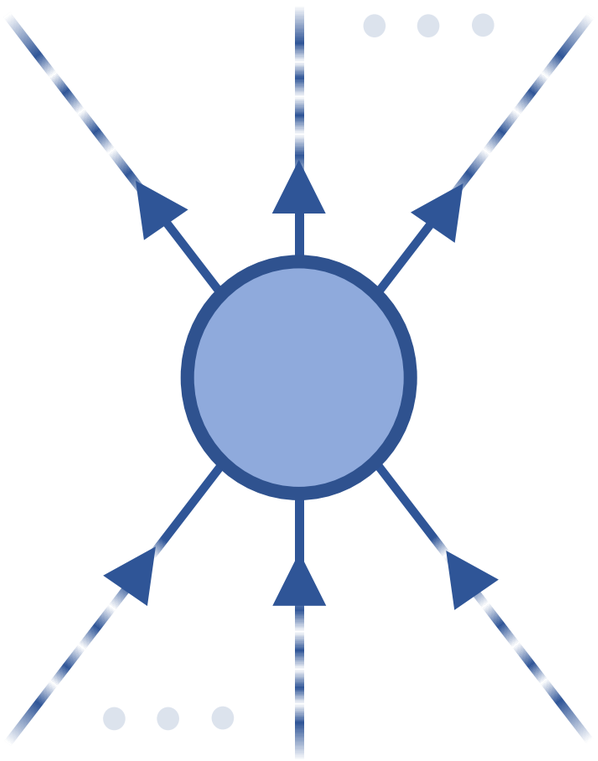


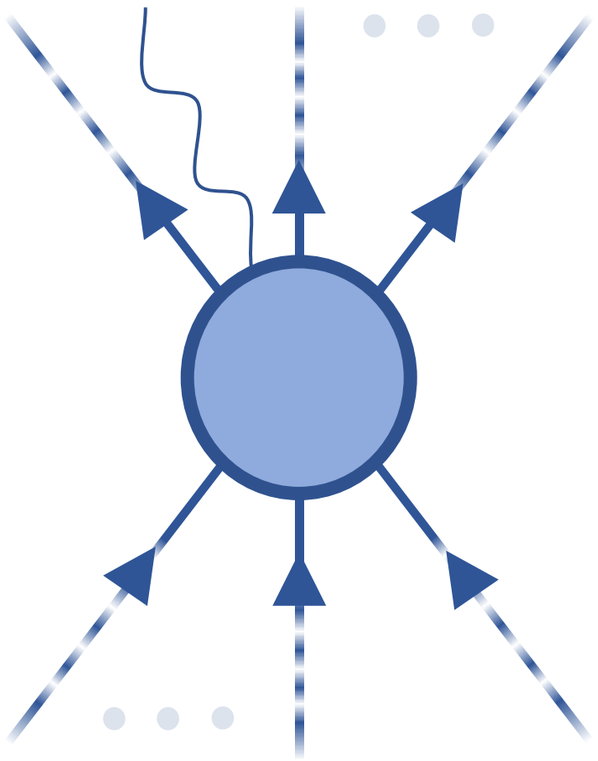
Ward Identities from Soft Theorems



$\mathcal{S}$-matrix elements are typically computed between in and out states of particles with definite momentum p^{μ}

Soft theorems describe a relation between a given scattering process and one with an extra gauge particle as its energy approaches zero

Ward Identities from Soft Theorems



$$\langle out; \omega\, \hat{q}, \pm | \mathcal{S} | in \rangle = (S^{(0)} + \omega S^{(1)})\, \langle out | \mathcal{S} | in \rangle + \cdots$$

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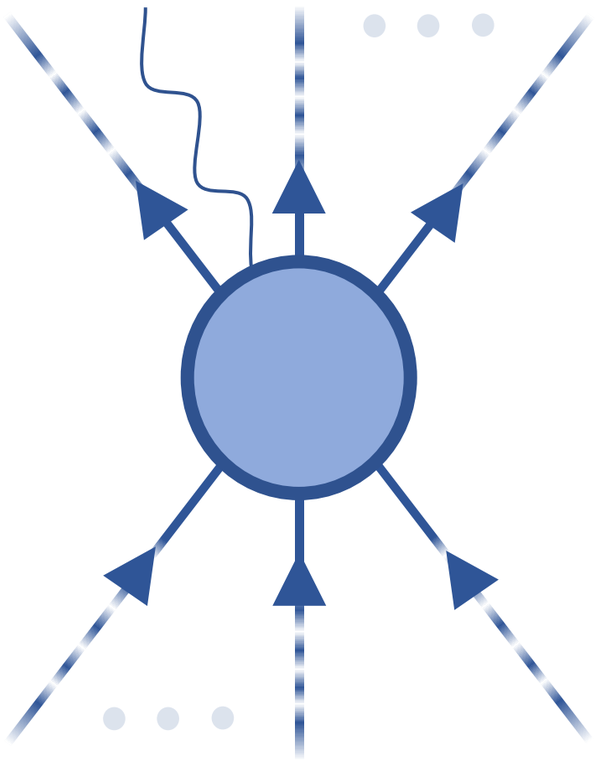
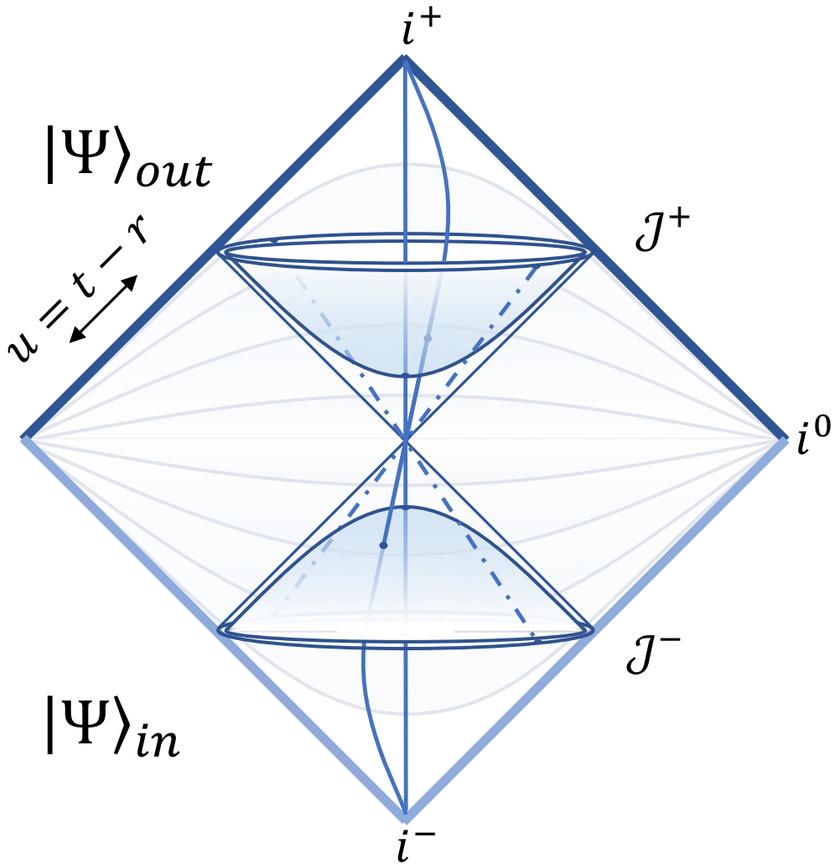


we can connect

$\hat{q}_{pos} \Leftrightarrow \hat{q}_{mom}$

and

$\lim_{\omega \to 0} \Leftrightarrow \int du$



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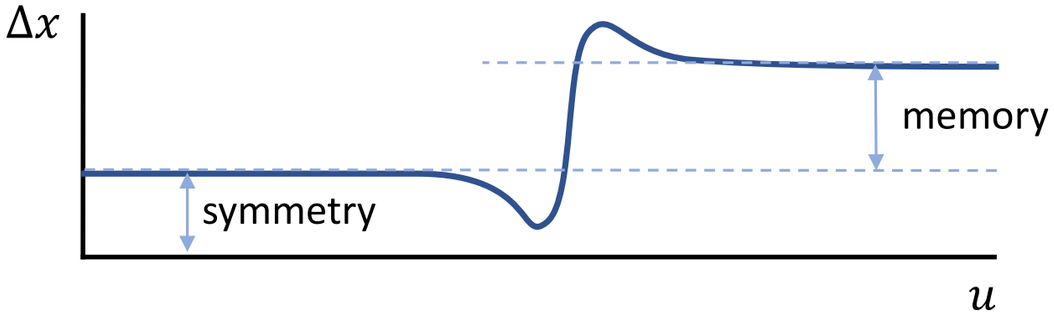
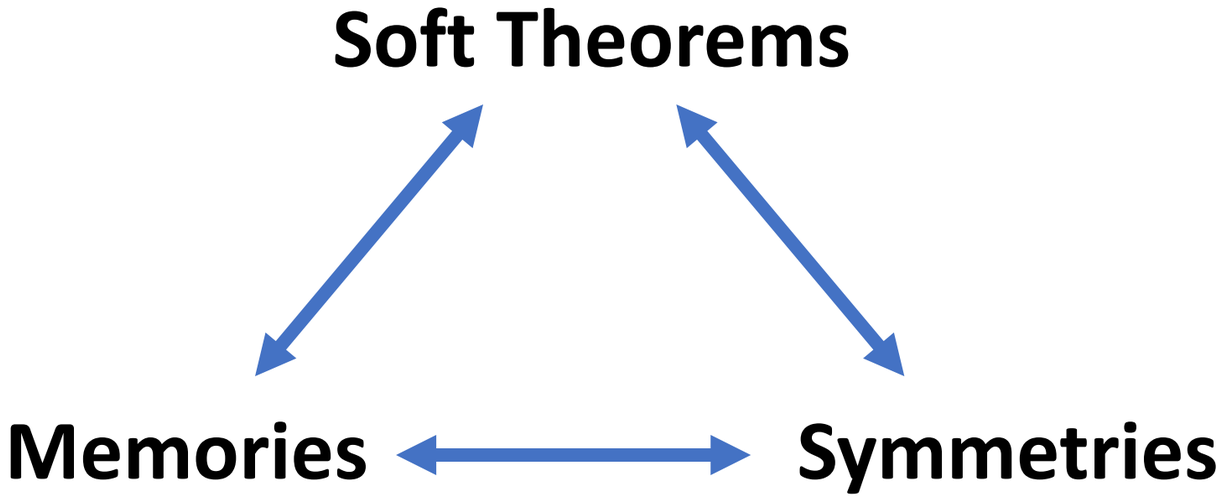
Ward Identities from Soft Theorems

- Strominger [arXiv:1308.0589 arXiv:1312.2229]:
  - Soft theorems can be used to construct Ward identities
  - Large gauge transformations S^2 dependent rather than constant

✓ Modus Operandi:
  - Look for larger set of ``physical" symmetries
  - Motivate via properties of low energy scattering



A Triangle of Relations



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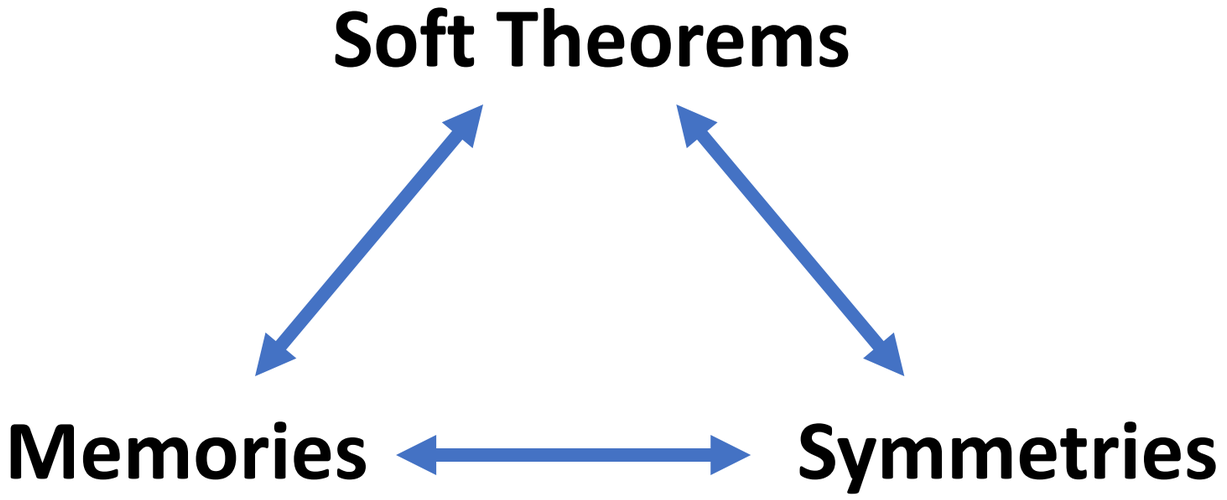

what we're after:
More Symmetries ⇒ More Constraints on $\mathcal{S}$-matrix

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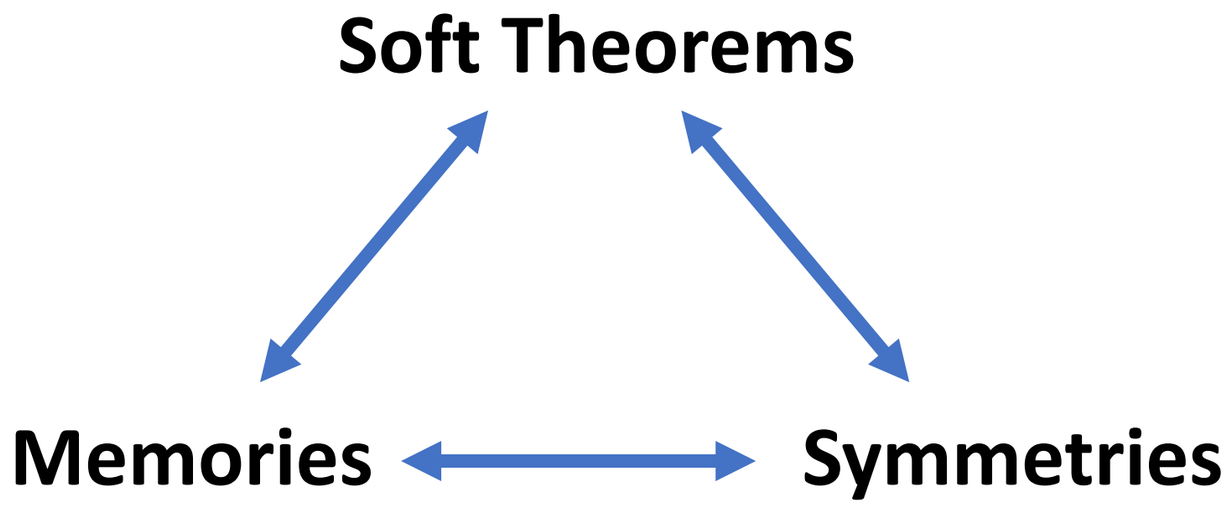

non-zero net effects in a typical scattering process
forces us to have asymptotic behavior that allows them,
these extra symmetries then act non-trivially

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relate $\mathcal{S}$-matrix elements for states with and without extra soft gauge particle

Soft Theorems

Memories ⟷ **Symmetries**

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Soft Theorems

Fourier transform:
long time ↔ low energy

Ward identities
[arXiv:1308.0589 arXiv:1312.2229]

Memories ⟷ **Symmetries**

non-zero net effects in a typical scattering process forces us to have asymptotic behavior that allows them, these extra symmetries then act non-trivially

step (net change)
vs baseline (starting point)

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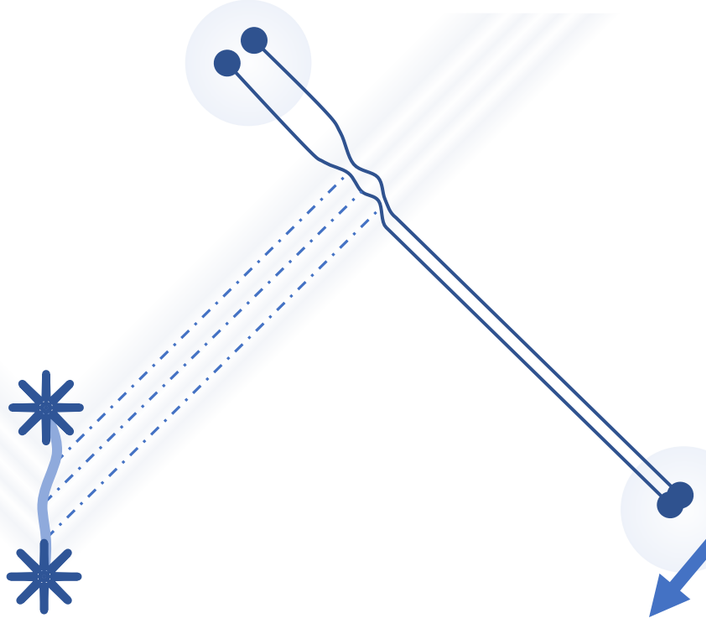

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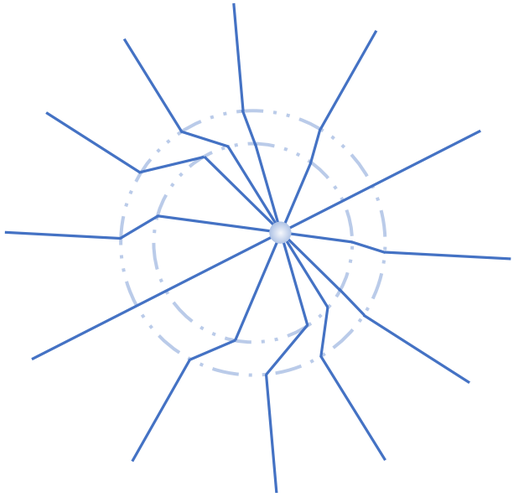
ii) Weinberg – graviton O($\frac{1}{\omega}$)
iii) Cachazo & Strominger – graviton O(1)

Soft Theorems



Memories

i) Liénard-Wiechert / Bieri & Garfinkle
ii) Zeldovich & Polnarev / Christodoulou
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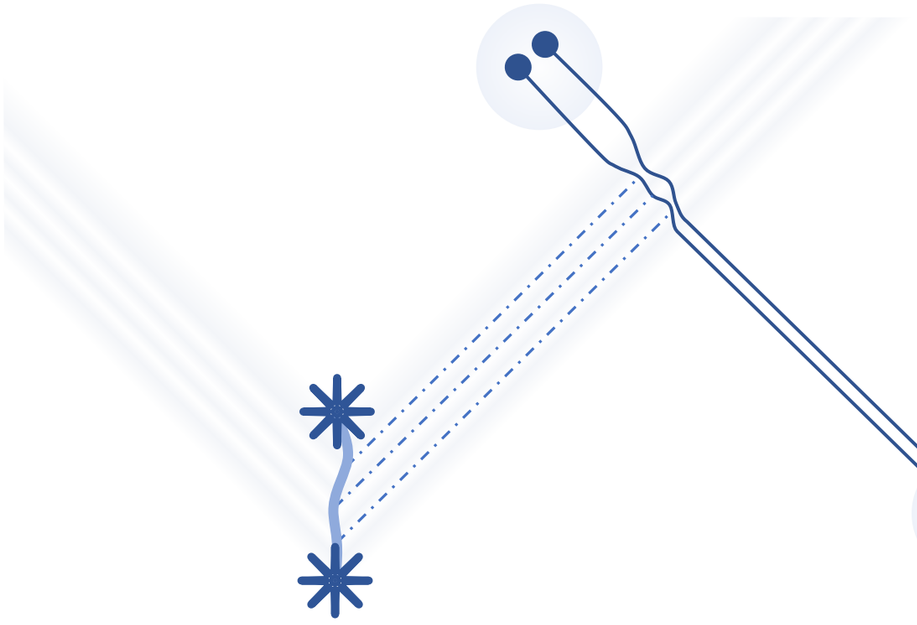
Symmetries

(global)	(asymptotic)
i) e-charge	large U(1)
ii) P_μ	supertranslations
iii) $J_{\mu\nu}$	superrotations

A Triangle of Relations



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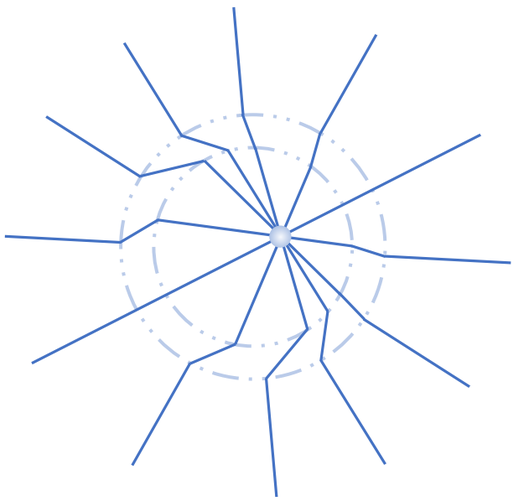

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Conformal Symmetries of the Celestial S^2

This triangle of relations guided the completion of a new iteration which has the structure of a Virasoro algebra on the celestial sphere

- Reparameterization invariance in Soft-Collinear Effective Theory
- Highest-weight scattering