Celestial Holography

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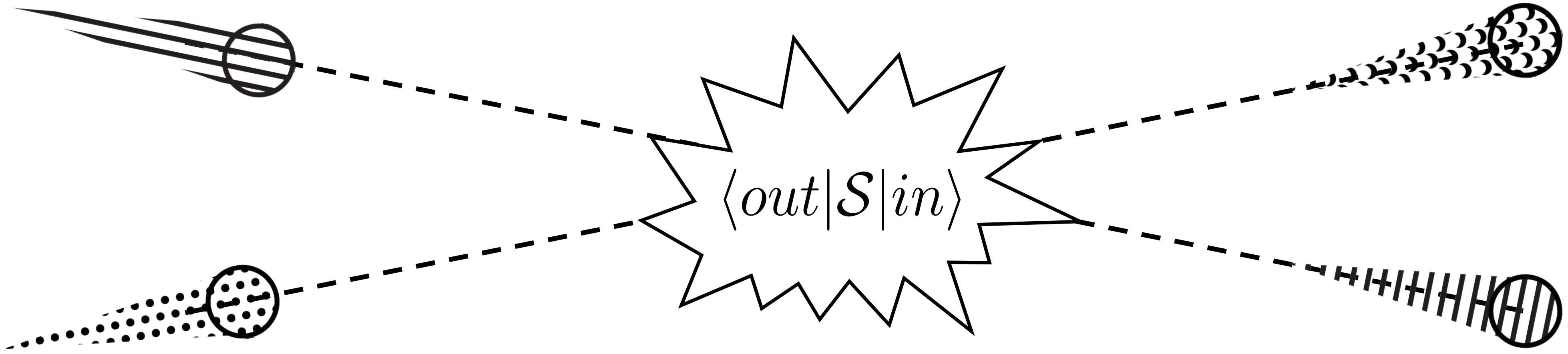
The Plot

Today, I will present a new framework to describe scattering: **Celestial Holography**.

The story will unfold in three acts which will address:

- Why **infrared physics** constrains scattering
- How symmetries of the S-matrix point to a **holographic dual**
- What we can discover by building this **dictionary**

The central object of study will be the **<u>S-matrix</u>** from which one can compute how likely a given scattering process is.

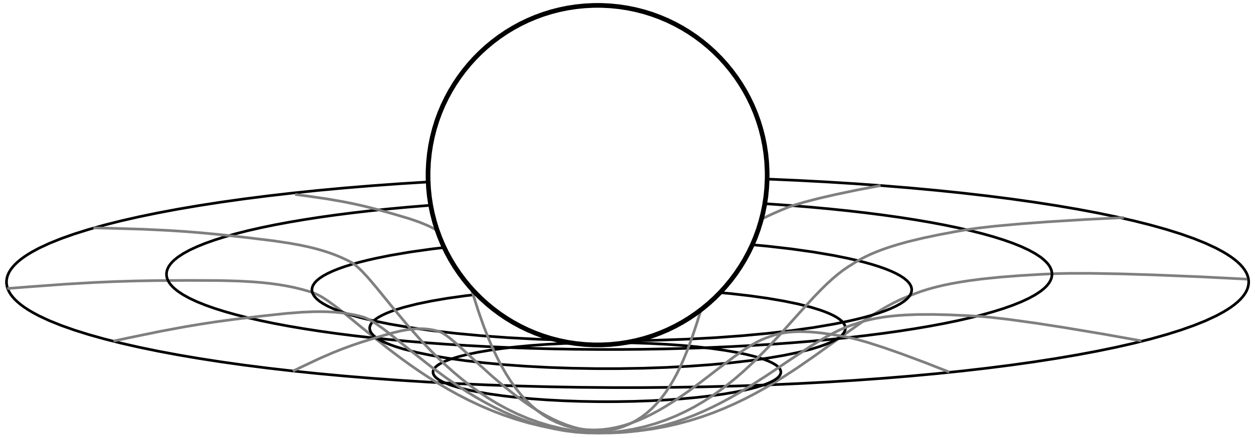


We will want our S-matrix to describe not only **<u>collider experiments</u>** but also **<u>gravitational scattering</u>**.

The Conflict

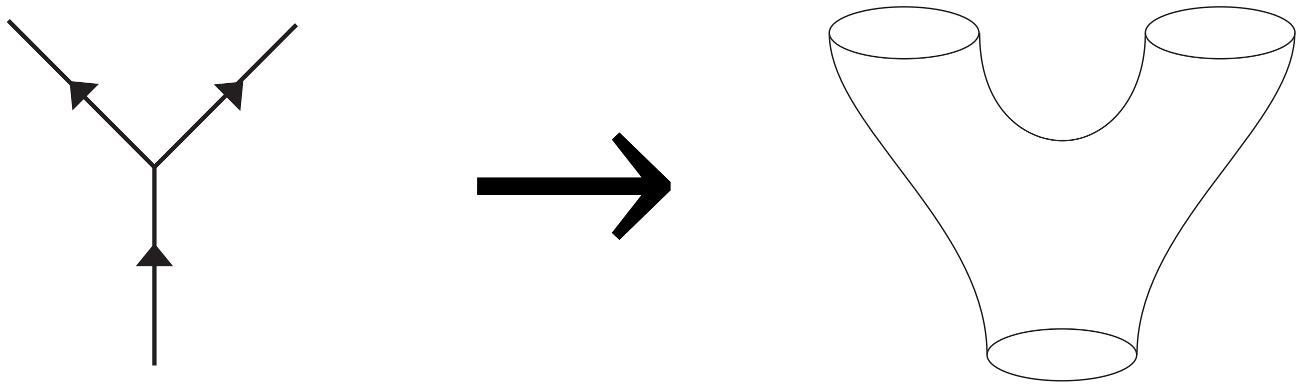
$$i\hbar \frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$$

+



Attempts at combining quantum mechanics and general relativity run into troubling issues, such as **<u>non-renormalizability</u>** and the **<u>information paradox</u>**.

One could try to resolve this puzzle by formulating a different description of physics in the **ultraviolet**.



Perhaps, surprisingly, we can learn a lot about quantum gravity starting from the **infrared**.

The Heroine

Our main tools will be **<u>Noether's theorem</u>**:

More Symmetries ⇒ More Constraints

and the **<u>holographic principle</u>**:

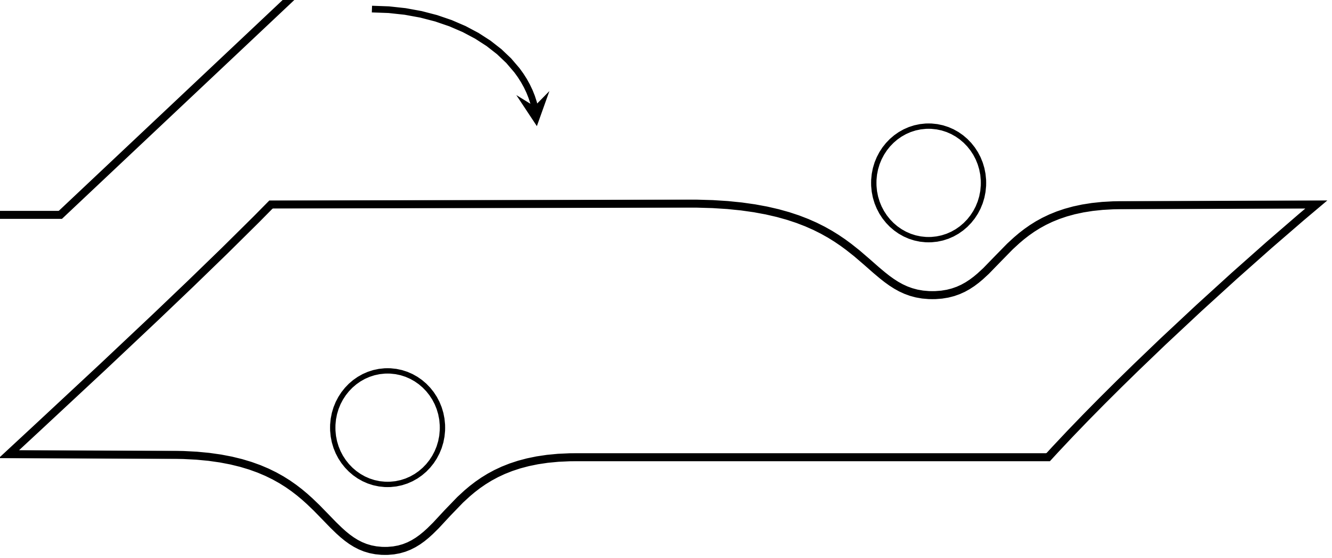
A theory of **quantum gravity** is equivalent to some lower dimensional theory **without gravity**.

Act I: Secrets in the Infrared

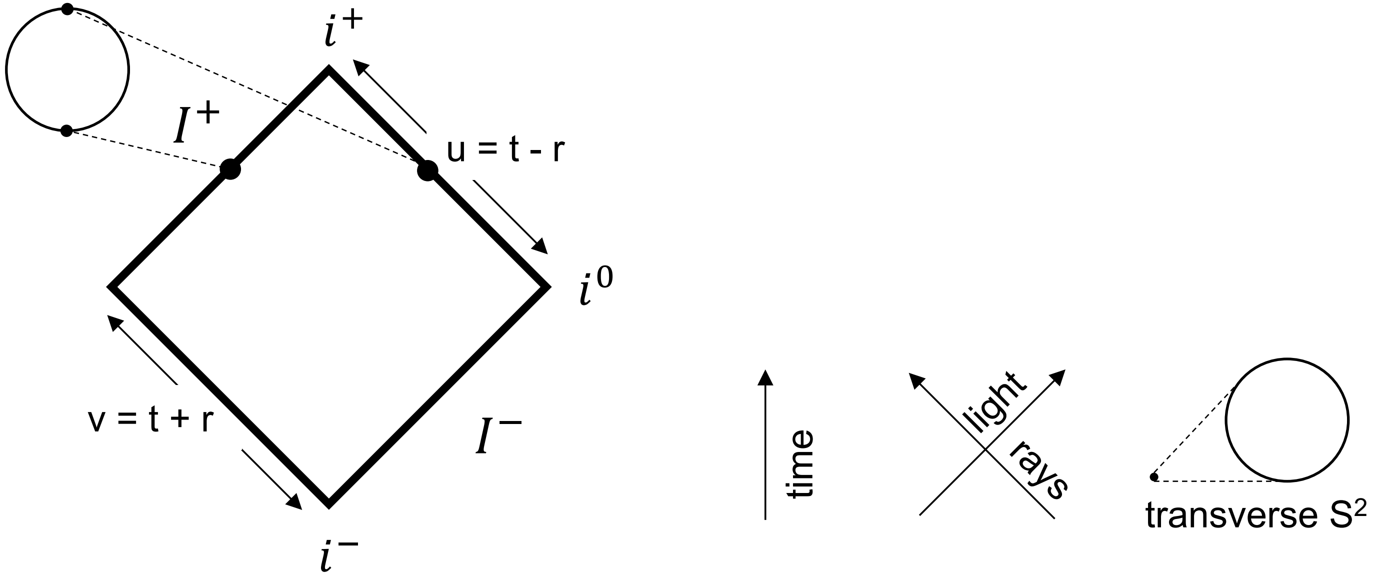
The Setting

We are interested in the class of spacetimes with $\mathbf{\Lambda = 0}$ allowing **localized stress tensor sources**.

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$



These spaces have the same **causal structure** as Mink_4.



In particular, m=0 fields enter and exit along **null boundaries**.

$$I^{\pm} \cong \mathbb{R} \times S^2$$

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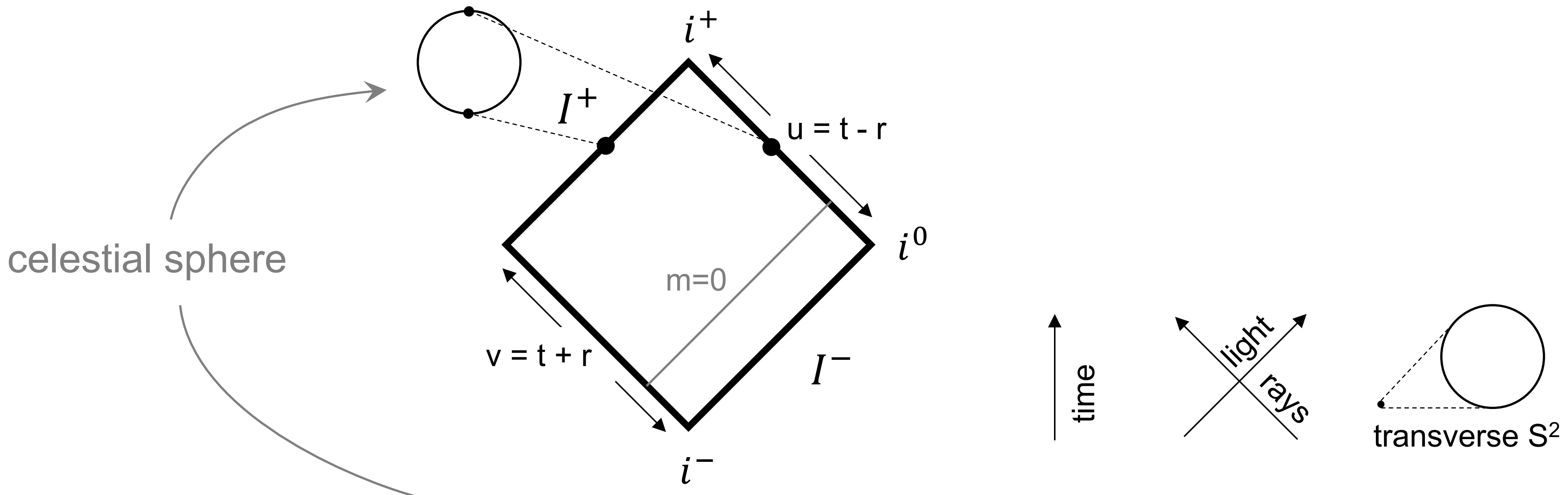
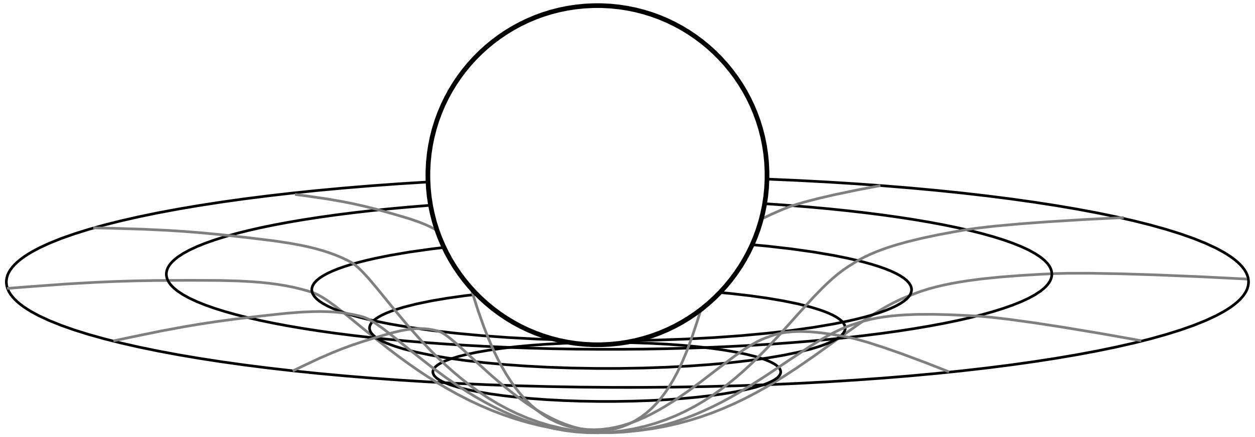


In particular, m=0 fields enter and exit along **null boundaries**.

$$I^{\pm} \cong \mathbb{R} \times S^2$$

From Newtonian gravity, we expect **long range** deformations of the metric once we add matter.

$$\Phi = -\frac{GM}{r}$$



For **asymptotically flat spacetimes**, we can perform a **large-r expansion** near null infinity.

The metric near future null infinity can be parameterized as

$$ds^2 \quad = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} + 2\frac{m_B}{r}du^2$$
$$+\big(rC_{zz}dz^2 + D^zC_{zz}dudz + \frac{1}{r}(\frac{4}{3}N_z - \frac{1}{4}\partial_z(C_{zz}C^{zz}))dudz + c.c.\big) + ...$$

Radiative Data

These falloffs are preserved by the following diffeomorphisms

$$\xi^+ = (1+\frac{u}{2r})Y^{+z}\partial_z - \frac{u}{2r}D^{\bar{z}}D_zY^{+z}\partial_{\bar{z}} - \frac{1}{2}(u+r)D_zY^{+z}\partial_r + \frac{u}{2}D_zY^{+z}\partial_u + c.c$$
$$+ f^+\partial_u - \frac{1}{r}(D^zf^+\partial_z + D^{\bar{z}}f^+\partial_{\bar{z}}) + D^zD_zf^+\partial_r$$

Superrotations

Supertranslations

$$z = e^{i\phi}\tan\frac{\theta}{2} \quad \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$
$$f^+ = f^+(z,\bar{z}) \qquad \partial_{\bar{z}}Y^{+z} = 0$$

[Bondi, van de Burg, Metzner ‘62] [Sachs ‘62][Barnich, Troessaert ‘11]

Diffeomorphisms which act non-trivially on the boundary data are considered part of the **asymptotic symmetry group**.

$$\text{ASG} = \frac{\text{allowed gauge symmetries}}{\text{trivial gauge symmetries}}$$

These transformations give a **non-zero canonical charge**.

We see that this class of metrics obeys an **infinite dimensional** enhancement of Poincaré, which includes:

- **supertranslations** which shift the u-coordinate by a function of $(z, \bar{z})$

$$\xi^+|_{\mathcal{I}^+} = f^+(z, \bar{z})\partial_u$$

- **superrotations** which extend the global Lorentz transformations

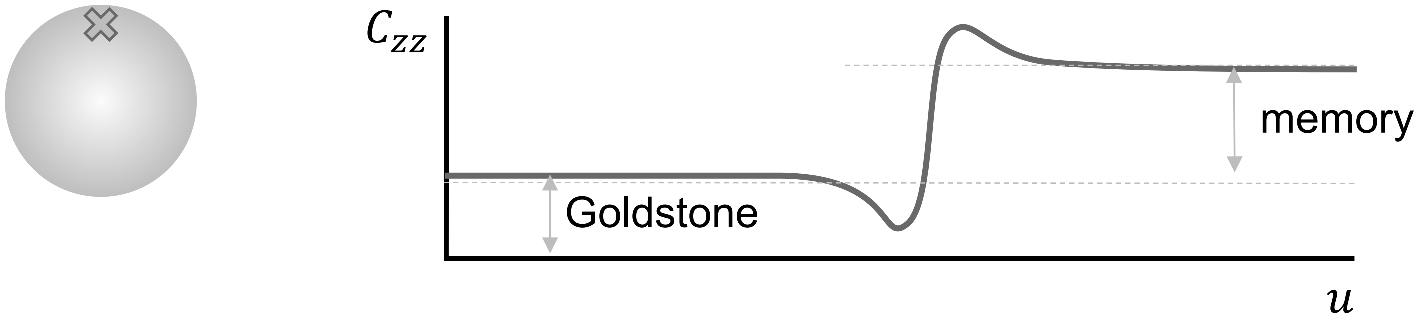
$$Y^z_{12} = iz, \quad Y^z_{13} = -\tfrac{1}{2}(1 + z^2), \quad Y^z_{23} = -\tfrac{i}{2}(1 - z^2),$$
$$Y^z_{03} = z, \quad Y^z_{01} = -\tfrac{1}{2}(1 - z^2), \quad Y^z_{02} = -\tfrac{i}{2}(1 + z^2).$$

  to local CKV's $Y^z(z)$.

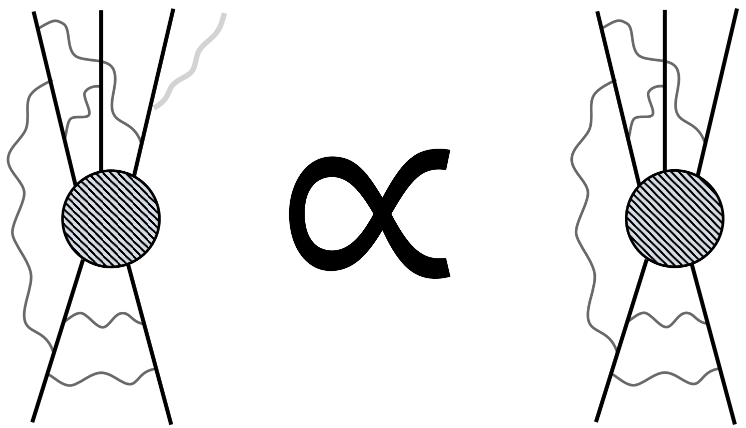
The IR dynamics of gravity is governed by the spontaneous breaking of these **asymptotic symmetries**.

For every **Goldstone mode** of an asymptotic symmetry, we expect a conjugate **memory mode**.

These memory modes are observable and provide a way to **experimentally test** the proposed ASG.

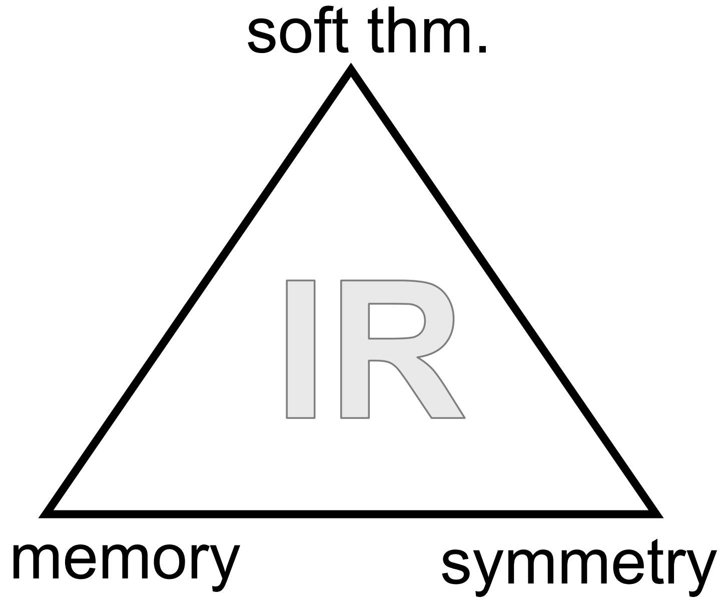


Meanwhile, one can verify that the **S-matrix** obeys these symmetries by translating these statements to **momentum space**.



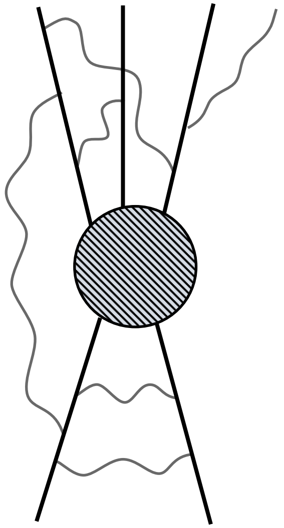
It turns out that the **Ward identities** for asymptotic symmetries are equivalent to **soft theorems** in QFT.

We see a **pattern of connections** between low energy observables, symmetries, and soft theorems emerging in the infrared.



This Infrared Triangle is **universal** and can be used as a template to look for **new physics**.

Let us consider an example where each vertex was **new**.



$$\langle out|a_-(q)\mathcal{S}|in\rangle = \left(S^{(0)-} + S^{(1)-}\right)\langle out|\mathcal{S}|in\rangle + \mathcal{O}(\omega)$$

$$S^{(0)-} = \sum_k \frac{(p_k \cdot \epsilon^-)^2}{p_k \cdot q} \qquad S^{(1)-} = -i\sum_k \frac{p_{k\mu}\epsilon^{-\mu\nu}q^\lambda J_{k\lambda\nu}}{p_k \cdot q}$$

The **soft theorem** relating amplitudes with and without an extra low energy graviton can be extended to subleading order.

[Weinberg ‘65] [Cachazo, Strominger ‘14]

Meanwhile, the charges generating superrotations

$$8\pi G Q^{+}[Y] = \int_{\mathcal{I}^{+}} \sqrt{\gamma} d^{2}z du \left[-\frac{1}{2} D_z^3 Y^z u \partial_u C^{zz} + Y^z T_{uz} + u D_z Y^z T_{uu} + h.c. \right]$$

$$Q^{+}[Y] = Q_S^{+}[Y] + Q_H^{+}[Y]$$

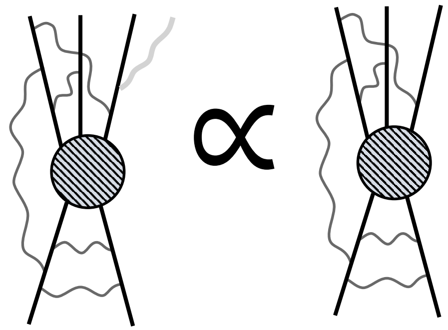
split into a part linear in the metric Q_S and a part measuring the stress tensor flux along null infinity Q_H.

[Kapec, Lysov, SP, Strominger '14]

Q_S picks out a certain low energy mode of the metric

$$\int du u \partial_u C_{\bar{z}\bar{z}} = \frac{i\kappa}{8\pi} \hat{\epsilon}_{\bar{z}\bar{z}} \lim_{\omega \to 0} (1 + \omega \partial_\omega)[a_-(\omega \hat{x}) - a_+(\omega \hat{x})^\dagger]$$

whose S-matrix insertions are given by the soft theorem.



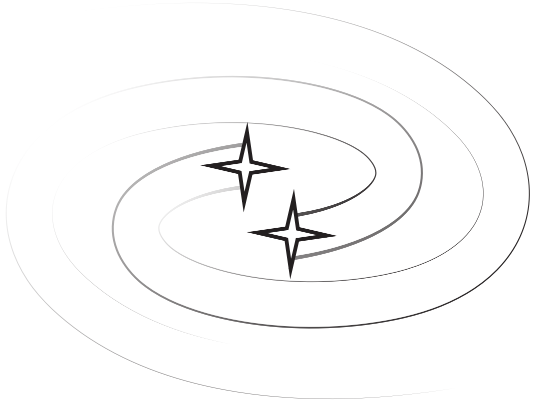
$$\Leftrightarrow \quad \langle out | Q^+[Y] \mathcal{S} - \mathcal{S} Q^-[Y] | in \rangle = 0$$

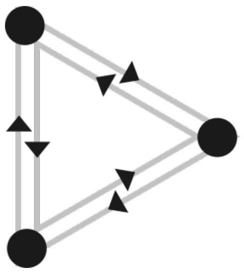
[Kapec, Lysov, SP, Strominger '14]

We see that the S-matrix obeys a **Virasoro symmetry** which lets us **independently** boost and rotate different patches of the night sky.

This appears to be **phenomenologically relevant** to Jet physics via the reparameterization invariance of SCET.

Moreover, if we wanted to test the astrophysical relevance of this symmetry group, one could measure **spin memory**.





This is a Sagnac-like effect measuring angular momentum flux through your interferometer.

[SP, Strominger, Zhiboedov '15]

The Intermission

The example I just covered is the stepping off point into the wonderful world of Celestial Holography.

Before continuing our journey, I want to emphasize the power of what we have already gained from the structure of IR physics.

IR Divergences:

Our story offers a reinterpretation of IR divergences in terms of **charge (non)-conservation**. Rather than compute inclusive cross-sections, one can use dress states to enforce the Ward identity.

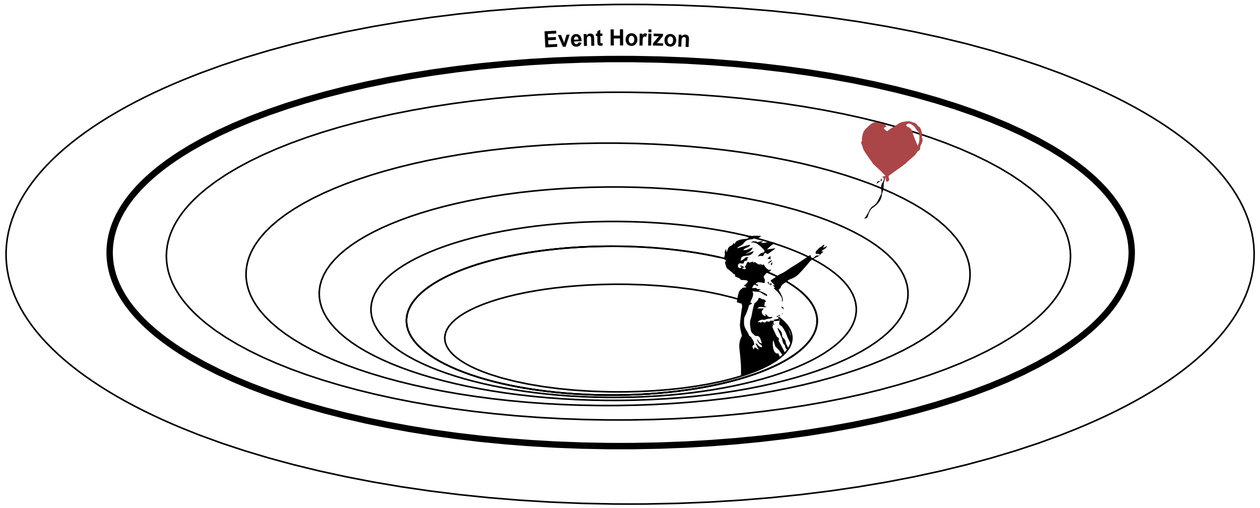
Black Hole Evaporation:

If you demand these symmetries hold in the presence of black holes, they constrain their evaporation and provide a source of **soft hair**.

[Chung ‘65] [Faddeev, Kulish ‘70] [Kapec, Perry, Raclariu, Strominger ‘17]

[Hawking, Perry, Strominger ‘16]

These insights open up interesting research avenues. For example, one can use them to study the experience of an infalling observer.



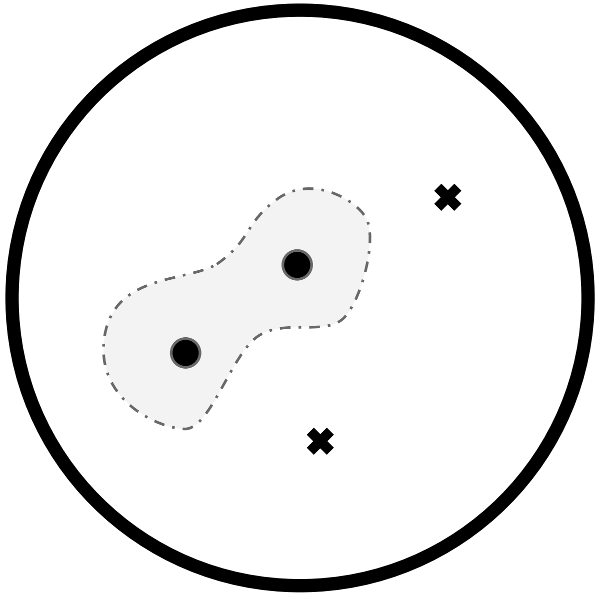
Her entanglement with soft degrees of freedom prevents her from experiencing a firewall!

[SP, Verlinde ‘20]

Act II: A New Framework for Scattering

The example we have examined suggests that the **4D S-matrix** is dual to a **2D Celestial CFT** living on the **Celestial Sphere**.

$$\langle out|\mathcal{S}|in\rangle =$$

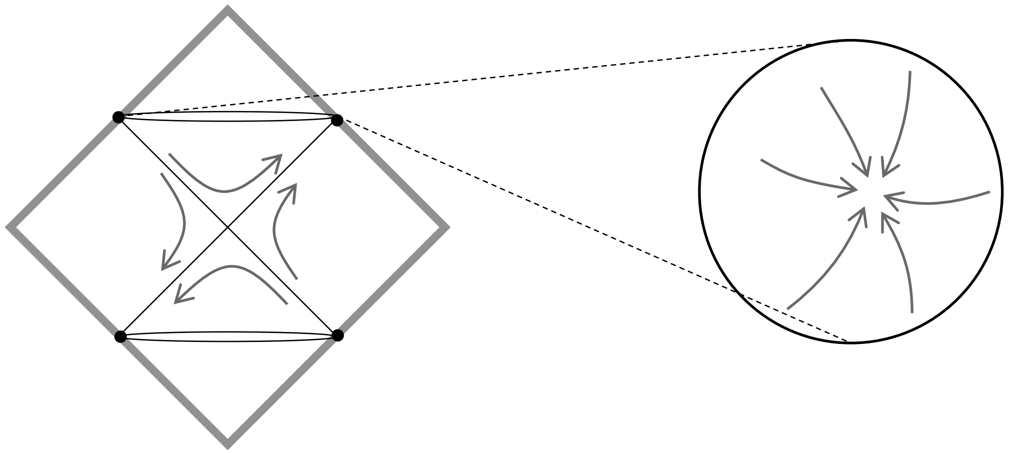


[SP, Shao ‘17] [SP, Shao, Strominger ‘17]

The global part of the conformal symmetry comes as no surprise.

$$\eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \Lambda^{\mathsf{T}} \eta \Lambda = \eta$$

Lorentz transformations induce conformal transformations on the celestial sphere.



However, with superrotations, we also have a Virasoro symmetry. For a particular choice of Y^z

$$T_{zz} = 2iQ_S^+(Y^w = \frac{1}{z-w}, Y^{\bar{w}} = 0)$$

the **4D superrotation Ward identity** takes the form of a 2D stress tensor **conformal Ward identity**.

$$\langle T_{zz}\mathcal{O}_1 \cdots \mathcal{O}_n \rangle = \sum_{k=1}^{n} \left[\frac{h_k}{(z-z_k)^2} + \frac{\Gamma^{z_k}_{z_k z_k}}{z-z_k} h_k + \frac{1}{z-z_k}\left(\partial_{z_k} - |s_k|\Omega_{z_k}\right) \right] \langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle$$

[Kapec, Mitra, Raclariu, Strominger ‘16]

We see that the relationship between soft theorems and Ward identities is much richer.

subleading soft graviton theorem = Ward identity for 4D superrotations = Ward identity for 2D stress tensor

(subleading soft graviton theorem = Ward identity for 4D superrotations): the asymptotic symmetry is physical

[Kapec, Lysov, SP, Strominger ‘14]

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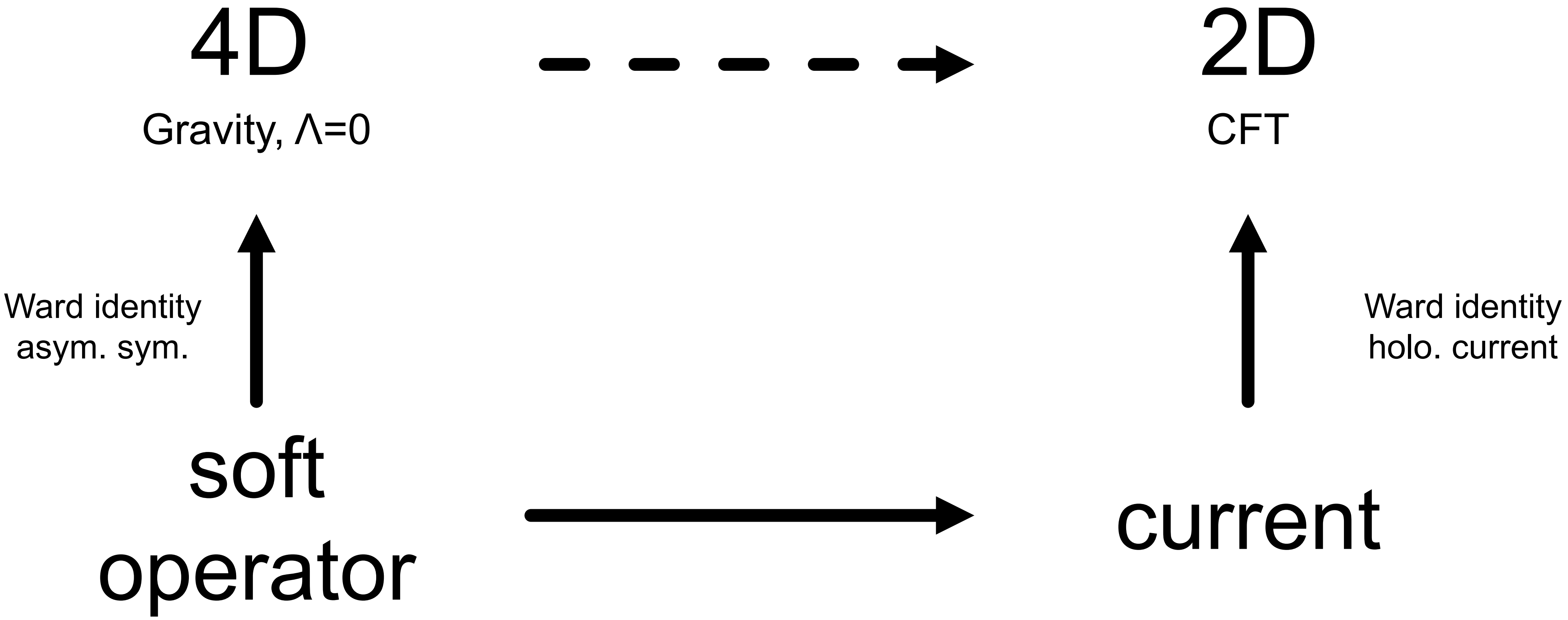
subleading soft graviton theorem = Ward identity for 4D superrotations = Ward identity for 2D stress tensor

we should look for a 2D dual CFT

S-matrix → cCFT correlator



We want to extend this map on the currents to send **any** S-matrix element to a conformal correlator.

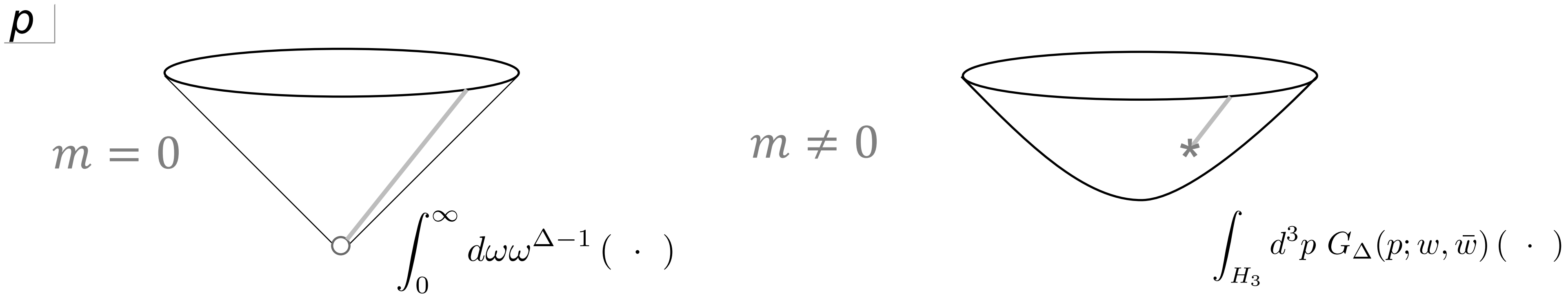
$$\langle out|\mathcal{S}|in\rangle \mapsto \langle \mathcal{O}_1 ... \mathcal{O}_n\rangle$$

In writing the superrotation Ward identity as a stress tensor OPE, they were a bit hasty.

$$h_k = \frac{1}{2}(s_k - \omega_k \partial_{\omega_k}), \quad \bar{h}_k = \frac{1}{2}(-s_k - \omega_k \partial_{\omega_k})$$

The conformal weights appearing there were still differential operators, not **diagonalized** in the plane wave basis.

To achieve this, we need to switch from **translation eigenstates** to **boost eigenstates**.



This amounts to constructing **wavepackets of on-shell states** that transform under $SL(2,\mathbb{C})$ with definite weight Δ and spin J.

[SP, Shao '17] [SP, Shao, Strominger '17]

Up to an overall normalization, the Mellin transformed packets are gauge equivalent to **conformal primary wavefunctions**:

$$h^{\Delta,\pm}_{\mu\nu;a}\left(\Lambda^{\mu}{}_{\nu}X^{\nu};\frac{aw+b}{cw+d},\frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}\right)=(cw+d)^{2h}(\bar{c}\bar{w}+\bar{d})^{2\bar{h}}\Lambda_{\mu}{}^{\rho}\Lambda_{\nu}{}^{\sigma}h^{\Delta,\pm}_{\rho\sigma;a}(X^{\mu};w,\bar{w})$$

We can construct a 1:1 map between single particle states and local cCFT operators with weights on the principal series.

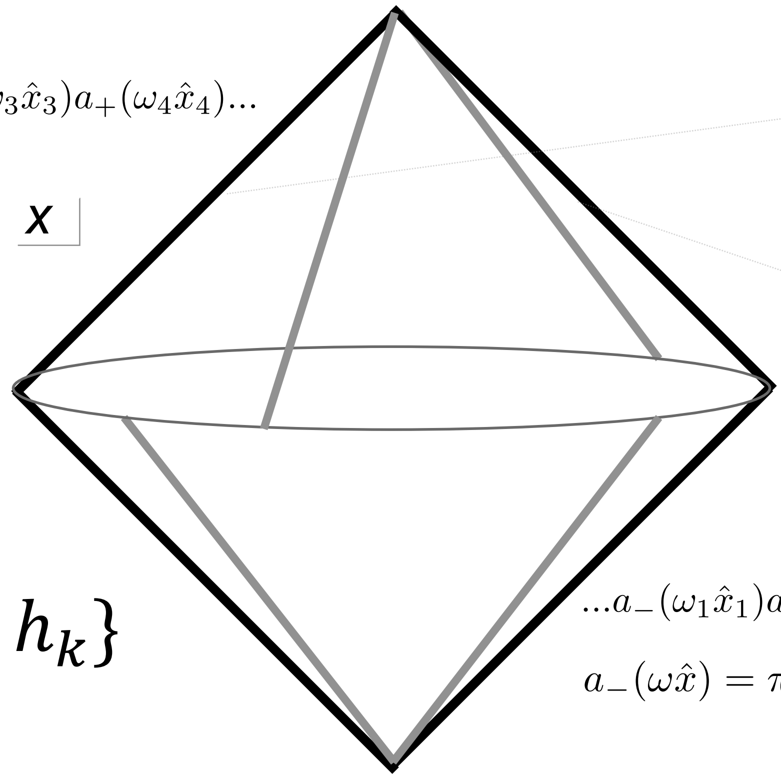
$$\Delta\in 1+i\mathbb{R}$$

We call this the **conformal basis**.

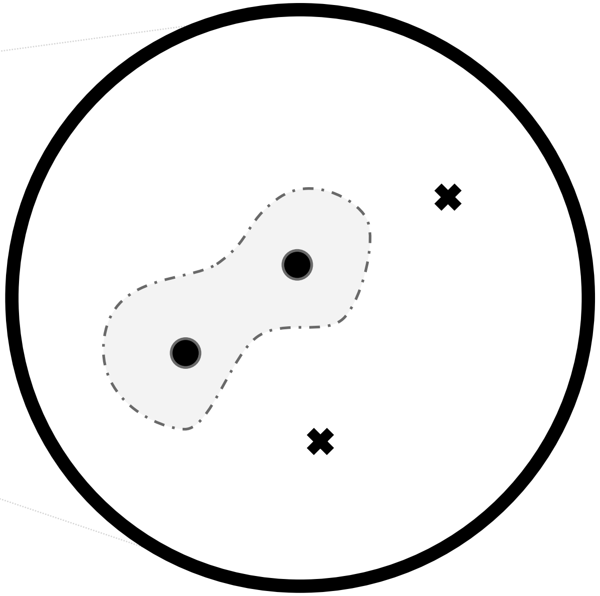
[SP, Shao '17]

$(h,\bar{h})=\frac{1}{2}(\Delta+J,\Delta-J)$

By writing m=0 S-matrix elements as 4D correlators of light ray operators (vs LSZ), we see an extrapolate dictionary.



$$a_-(\omega \hat{x}) = \pi i (1 + z\bar{z})^2 \int_{-\infty}^{\infty} du e^{i\omega u} C_{\bar{z}\bar{z}}(u, \hat{x})$$



The u direction is traded for a continuous spectrum of weights.

If you hand me an **amplitude**, I can then do an integral transform to tell you the corresponding **Celestial CFT correlator**.

$$\widetilde{\mathcal{A}}(\Delta_i, \vec{w}_i) \equiv \prod_{k=1}^{n} \int_0^\infty d\omega_k \omega_k^{\Delta-1} \, \mathcal{A}(\pm\omega_k q_k^\mu)$$

This transform probes scattering at **all energy scales**, puts the IR **symmetry enhancement front and center**, and gives a **new vantage point** to examine fundamental features of scattering.

[SP, Shao ‘17] [SP, Shao, Strominger ‘17]

Act III: Prospects for this Framework

Let’s explore what Celestial Holography can do for you and what **<u>you</u>** can do for Celestial Holography!

We are ultimately united by our interest in fundamental questions:

1. What are the **<u>symmetries</u>** of nature?
2. How do **<u>black holes</u>** process quantum information?
3. What is the ultimate **<u>UV</u>** description of our universe?

1. We’ve seen how the **IR triangle** helps us identify **new symmetries** of the S-matrix. ✓

2. We have also pointed out that these symmetries **constrain black hole evaporation** and that the soft sector of phase space is relevant to the experience of an infalling observer. ✓

3. We will now turn to open and active questions surrounding what makes for a **consistent celestial amplitude** and how this might constrain the UV behavior of scattering.

[SP, Strominger, Zhiboedov ‘15] [SP ’15] [SP, Verlinde ‘20]

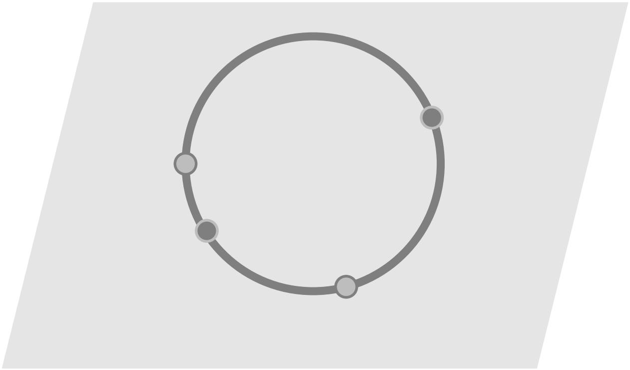
The Theme

There are two (overlapping) approaches one can take

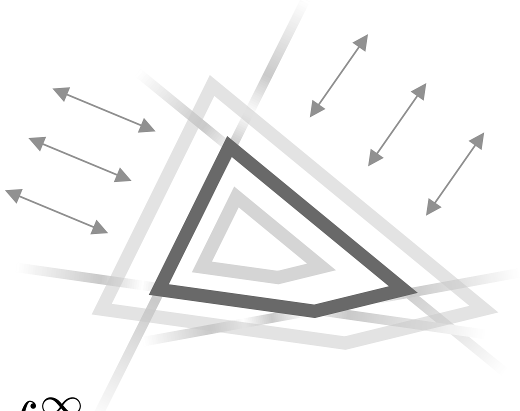
a) Complete the dictionary
   i. How are CFT features encoded in amplitudes?
   ii. How are amplitudes features encoded in cCFT?
   iii. Can we classify cCFTs?

b) Connect to other subfields
   i. Amplitudes
   ii. Bootstrap
   iii. Twistors
   iv. LQG

One notices right away that **Celestial CFTs** are quite exotic:



$$\mathrm{Im}\frac{z_{12}z_{34}}{z_{13}z_{24}} = 0$$



$$\tilde{\mathcal{A}} \propto \int_0^\infty ds s^{i\sum\lambda_k - 1} = 2\pi\delta(\sum \lambda_k)$$

Aside from the fact that conformal dimensions are complex, translation invariance gives strange singularities in the cross ratios and distributional dependence on the conformal dimension.

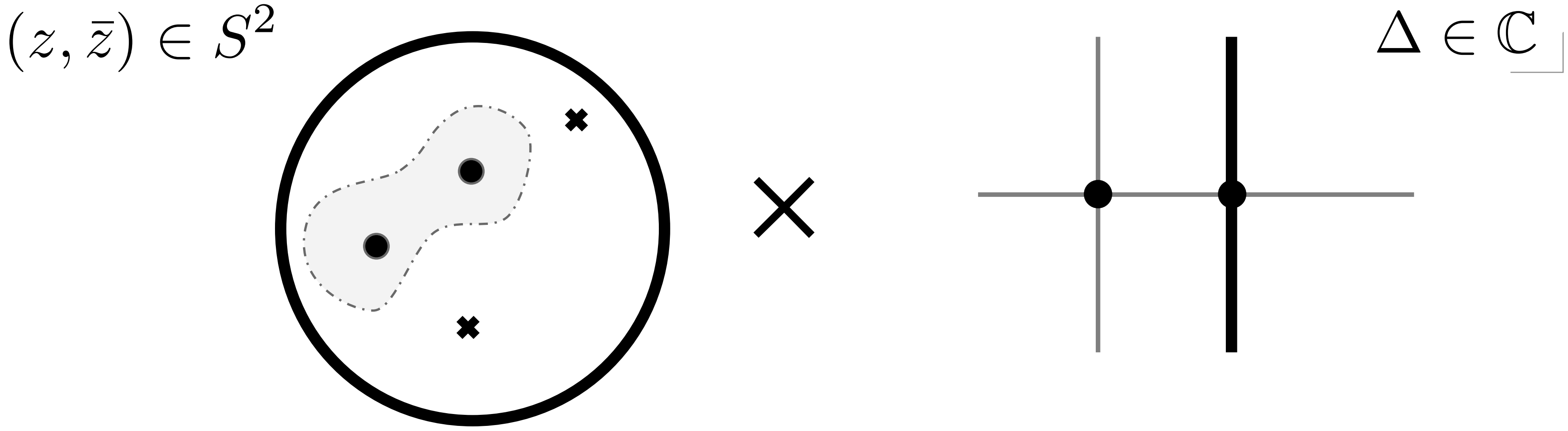
[SP, Shao '17] [SP, Shao, Strominger '17]

The Ideology

If we stick to the mindset that we **have** some exotic CFT and are willing to re-derive things we might take for granted about 2D CFTs, we see lovely results.

In particular, it is very useful to progress from transforming **particular amplitudes** to transforming **generic features** of amplitudes.

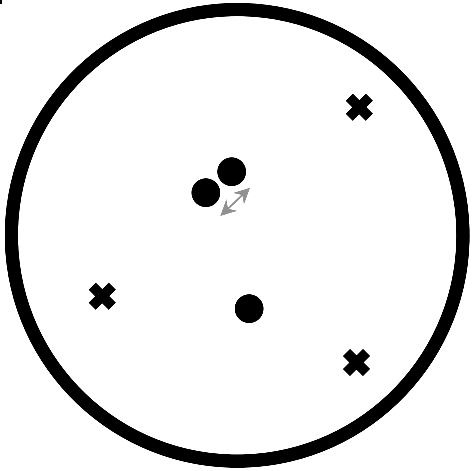
Celestial amplitudes are described by points on the celestial sphere and weights in a complex plane.



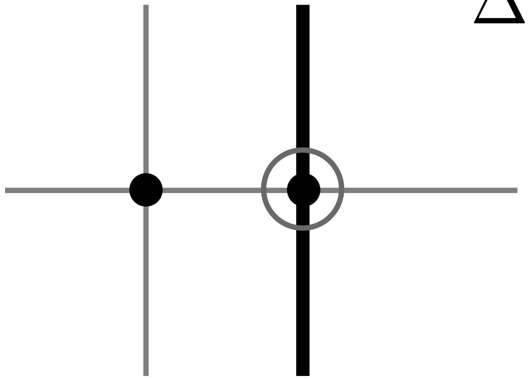
One of the first things to ask is what happens at special values of the z_i and Δ_i.

One sees that collinear limits are captured by Celestial OPEs and factorizations at special Δ probe (sub)leading soft theorems.

$(z, \bar{z}) \in S^2$



$\Delta \in \mathbb{C}$



Moreover, stringy UV behavior tames the Mellin transform!

[Stieberger, Taylor ‘18] [Fan, Fotopolus, Taylor ‘19] [Pate, Raclariu, Strominger, Yuan ‘19] [Puhm ‘19] [Guevara ‘19]

The list goes on!

- **ASG questions** can be translated to the conformal basis
- **Double Copy** relations persist for celestial amplitudes
- **Null state** relations constrain celestial CFT correlators

And there are many open questions…

[Banerjee, Pandey, Paul ‘19][Donnay, SP, Puhm ‘20] [Casali, Puhm ‘20] [SP, Puhm ‘20] [SP, Puhm, Trevisani]

The Cliffhanger

What do factorization channels look like for celestial amplitudes?

Should we be going to (2,2)?

Can we define a Boo**s**tstrap program?

Can we make a nice connection to edge modes?

The Cast

4 lectures, 10 talks, 8 gong show presentations
282 registrants – 49 faculty, 93 postdocs, 140 students
223 active on Slack
207 attending school on Zoom, 281 delayed views on Slack
204 attending workshop on Zoom, 397 delayed views on Slack

Total Cost: $89.59

https://pcts.online https://hub.link/sGPoiFp