From: Yoni Kahn <ykahn at mit.edu>
Subject: **Journal club today: Sabrina Pasterski**
Date: April 15, 2015 7:43:47 AM CDT
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Cc: "Pasterski, Sabrina" <spasterski at fas.harvard.edu>

Good morning all,

Today at 4:15pm we will have Sabrina Pasterski talking about asymptotic Virasoro symmetries and gravitational memory, based on her work 1406.3312 and 1502.06120. Refreshments at 4pm as usual.

Yoni

Asymptotic Symmetries: Virasoro
1406.3312
1502.06120

Outline

Theory $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

d.o.f $\subset g_{\mu\nu}$

b.c.

(what can you do to d.o.f. & stay w/in)

sym ξ obeying/maintaining b.c. → still change some d.o.f.

relevant e.o.m

(what are the corr. dynamics)

charge → Q_ξ ← parameterized by ξ — generating change in states

matching in & out

Q → $\mathcal{S}$-matrix ward identity

consistency w/ mode insertion

⟺ soft factor

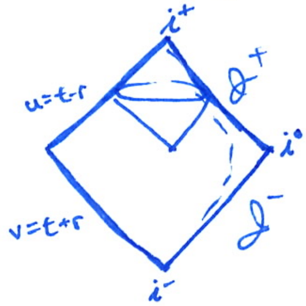
corresponding observable

⟺ memory

Conventions

Asymptotic symm. of radiation

$\Leftrightarrow$ limits of null traj.



$\mathbb{R}\times S^2$

$\uparrow$ use $z=e^{i\phi}\tan\frac{\theta}{2}$

$p^\mu = E(1,\hat{x})$ $m=0$

or $= m\gamma(1,\beta\hat{x})$ $m\neq 0$

$r\to\infty$ order of limits

$e^{iq\cdot x} = e^{-i\omega u - i\omega r(1-\hat{x}\cdot\hat{q})}$

large $r \longleftrightarrow$ small ω same $(z,\bar{z})$

$$g_{\mu\nu} = \begin{pmatrix} \frac{V}{r}e^{2\beta} + g_{AB}U^AU^B & -e^{2\beta} & -g_{BC}U^C \\ -e^{2\beta} & 0 & 0 \\ -g_{AC}U^C & 0 & g_{AB} \end{pmatrix} \qquad (u,r,z,\bar{z})$$

$\beta \sim r^{-2}$ $U^A \sim r^{-2}$ $\det g_{AB} = r^4\sin^2\theta$

$\uparrow$ doing analytic in $\frac{1}{r}$ expansion

what ξ leave $\mathcal{L}_\xi g_{\mu\nu} \sim \begin{pmatrix} \frac{1}{r} & \frac{1}{r^2} & \frac{1}{r^2} \\ \frac{1}{r^2} & 0 & 0 \\ \frac{1}{r^2} & 0 & r \end{pmatrix}$

$$ds^2 = -du^2 - 2du\,dr + 2r^2\underbrace{\gamma_{z\bar{z}}}_{\frac{2}{(1+z\bar{z})^2}}dz\,d\bar{z} + \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + D^zC_{zz}du\,dz + \frac{1}{r}\left(\frac{4}{3}N_z - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)du\,dz + \text{c.c.}$$

$\xi = f\partial_u - \frac{1}{r}D^Af\partial_A + D^zD_zf\partial_r$ $\leftarrow$ super translations

$+ \left(1+\frac{u}{2r}\right)Y^z\partial_z - \frac{u}{2r}D^{\bar{z}}D_zY^z\partial_{\bar{z}} - \frac{1}{2}(u+r)D_zY^z\partial_r + \frac{u}{2}D_zY^z\partial_u + \text{c.c.}$ $\leftarrow$ superrotations

$\uparrow$ meromorphic c.k.v

e.o.m $\begin{cases} \partial_u m_B = \frac{1}{4}\partial_u \left[D_z^2 C^{zz} + D_{\bar{z}}^2 C^{\bar{z}\bar{z}} \right] - T_{uu} \\ \partial_u N_z = \frac{1}{4}\partial_z \left[D_z^2 C^{zz} - D_{\bar{z}}^2 C^{\bar{z}\bar{z}} \right] + \partial_z m_B - T_{uz} \end{cases}$

$G_{\mu\nu} = 8\pi G T_{\mu\nu}$
r^{-2} term

$\int du \Leftrightarrow$ boundary terms

+ $\mathcal{I}^-$ part $\Rightarrow$ something to match across i^0

origin of $\langle \text{out} | Q^+ S - S Q^- | \text{in} \rangle = 0$ ward identity

have $Q = Q_S + Q_H$

Q_S: involves C_{zz}

Q_H: involves $T_{\mu\nu} \Leftrightarrow \mathcal{L}_\xi$ on hard particles

ex. for superrotations relevant, integrable part of charge is

$$8\pi G Q = \int \sqrt{\gamma}\, d^2z \left[u D_A Y^A m_B + Y^A N_A + \ldots \right]$$

use $\int du\, \partial_u [\quad]$ & e.o.m + i.b.p.

$\delta_Y C_{zz} = -u D_z^3 Y^z$ + hom. terms from $[\mathcal{L}_\xi g]_{zz}$

$[\cdot,\cdot]$ for $[\partial_u C_{zz}, \partial_{u'} C_{\bar{z}'\bar{z}'}]$

$\Rightarrow$ "Q_S^+" $= -\frac{1}{2} \int_{\mathcal{I}^+} du\, d^2z\, D_z^3 Y^z u \partial_u C^z{}_{\bar{z}}$

meanwhile

@ large r $\quad \xi \sim Y^z \partial_z + \frac{u}{2} D_z Y^z \partial_u$ + c.c. & conf. weight terms

$\Rightarrow_{\text{scalar}} \left(Y^z \partial_z - \frac{E_k}{2} D_z Y^z \partial_{E_k}\right) \langle \text{out} |$

"Q_H^+"

arrive at $\langle \text{out} | Q^+ S - S Q^- | \text{in} \rangle = 0$ Ward Identity

by having $Q = Q_S + Q_H$

$[\cdot,\cdot]$ $\quad \mathcal{L}_{\xi\,\text{mom. space}}$

need: matching (antipodal)

& soft factor

$\langle \text{out} | a_-(q) S | \text{in} \rangle = \left(S^{(0)-} + S^{(1)-}\right) \langle \text{out} | S | \text{in} \rangle + \mathcal{O}(\omega)$

$\int du\, u \partial_u \ldots$ picks out $\lim_{\omega \to 0} (1 + \omega \partial_\omega) \rightarrow S^{(1)}$

$S^{(1)} = -i \sum_k \frac{p_{k\mu} \varepsilon^{\mu\nu} q^\lambda J_{k\lambda\nu}}{p_k \cdot q}$

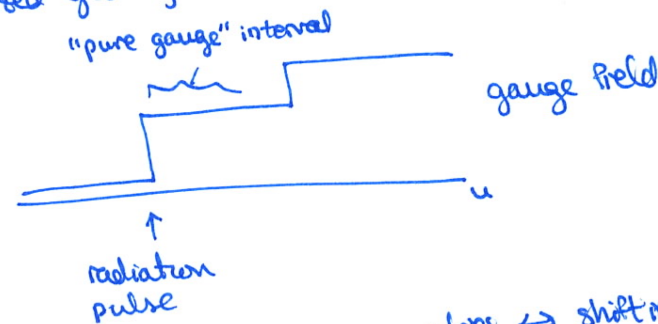
$\int d^2z\, D_z^3 Y^z$ convoluted w/ soft factor $\Rightarrow$ ✓ Ward identity have seen

asymptotic symmetries $\Longleftrightarrow$ soft factors

also

physical observables $\Longleftrightarrow$ soft factors

- mode expansion + $\int du$ of e.o.m $\Rightarrow$ $\omega \to 0$ observable "memory"
- linearized gravity supertranslations $\Longleftrightarrow$ analog of E&M $\int dt\, \vec{E}_{rad}$



- extra $(z, \bar{z})$ residual gauge symmetries $\Longleftrightarrow$ shifting the baseline / resetting the "vacuum"
- superrotations $\Longleftrightarrow$ subleading soft graviton factor $\Longleftrightarrow$ spin memory P.S.Z.