RPI and Superrotations

Sabrina Gonzalez Pasterski
(Dated: April 24, 2015)

I construct a superrotation vector field corresponding to matching local Lorentz transformations at a discrete set of points on the sphere. This connects the asymptotic symmetry group with RPI in SCET.

Infinitesimal superrotations are parameterized by meromorphic conformal killing vectors on the sphere at null infinity. This note is an exercise in matching a particular choice of Y^z to the constraint of having transformations at m well separated points locally look like arbitrary infinitesimal Lorentz transformations. The physical motivation behind this is that one can then match this asymptotic symmetry to reparameterization invariance (RPI) in SCET for m jet directions [1].

Consider the coordinate transformation:

$$z \to z + \epsilon(z). \tag{.1}$$

The standard infinitesimal Lorentz transformations correspond to $\epsilon(z) = a+bz+cz^2$ for some complex constants $\{a, b, c\}$, giving six real parameters for these global transformations.

Here, we would like to consider a particular $\epsilon(z)$ such that near z_i for $i \in \{1, ..., m\}$, it obeys the series expansion

$$\epsilon(z)|_{z\sim z_i} = a_i + b_i(z - z_i) + c_i(z - z_i)^2 + ... \tag{.2}$$

Each of the m points will be taken to be well separated from one another. (Note that the action of $\epsilon(z) = a + bz + cz^2$ can be rewritten as a quadratic of the form (.2) about each z_i to get $\{a_i, b_i, c_i\}$ in terms of $\{a, b, c\}$ and z_i – or vice versa by just expanding the first three terms in (.2) – allowing one to relate the local series expansion coefficients to equivalent global Lorentz parameters.)

An ansatz of the form:

$$\epsilon(z) = \sum_j^m (\alpha_j + \beta_j(z - z_j) + \gamma_j(z - z_j)^2) \left(\frac{\prod_{k\neq j}^m z - z_k}{\prod_{k\neq j}^m z_j - z_k} \right)^3 \tag{.3}$$

has the property that for $z = z_i + \delta$

$$\left(\frac{\prod_{k\neq j}^m z_i + \delta - z_k}{\prod_{k\neq j}^m z_j - z_k} \right)^3 \sim \delta^3 \quad i \neq j \tag{.4}$$

while it is $\mathcal{O}(1)$ for $i = j$. As such, a Taylor series expansion up to second order near any z_i only involves matching one term in the sum

$$a_i+b_i\delta+c_i\delta^2 = (\alpha_i+\beta_i\delta+\gamma_i\delta^2) \left(\frac{\prod_{k\neq i}^m z_i - z_k + \delta}{\prod_{k\neq i}^m z_i - z_k} \right)^3 \tag{.5}$$

up to $\mathcal{O}(\delta^2)$. Then we find

$$\begin{aligned}
\alpha_i =& a_i \\
\beta_i =& b_i - 3a_i \sum_{k\neq i}^m \frac{1}{z_i - z_k} \\
\gamma_i =& c_i - 3b_i \sum_{k\neq i}^m \frac{1}{z_i - z_k} \\
&+ 3a_i \left[2 \left(\sum_{k\neq i}^m \frac{1}{z_i - z_k} \right)^2 - \sum_{\substack{k,\ell\neq i \\ k>\ell}}^m \frac{1}{(z_i - z_k)(z_i - z_\ell)} \right]
\end{aligned} \tag{.6}$$

gives the desired set of $\{\alpha_i, \beta_i, \gamma_i\}$ in our ansatz (.3) to match (.2) near each z_i.

One then has $Y^z = \epsilon(z)$, which one can plug into the asymptotic form of the superrotation killing vector field [2]

$$\xi_Y = \frac{u}{2} D_a Y^a \partial_u + Y^a \partial_a + ... \tag{.7}$$

Locally, this will generate the specified Lorentz transformations at the m points on the sphere at null infinity. At the same time, RPI can be expressed as independent Lorentz transformations of each jet. Here, we think of parameterizing the collinear direction for each jet by z_i. Since massless scatterers end up at future null infinity, these local Lorentz transformations naturally correspond to RPI changes in the tetrad corresponding to each jet direction.

ACKNOWLEDGEMENTS

Many thanks to the attendees of the MIT LHC/BSM journal club for interesting questions.

[1] "Soft Theorems from Effective Field Theory," arXiv:1412.3108 [hep-th].

[2] "Semiclassical Virasoro Symmetry of the Quantum Gravity $\mathcal{S}$-Matrix," arXiv:1406.3312 [hep-th].