Speed Demon

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This note proposes a detector arrangement/measurement corresponding to the subleading soft graviton theorem.[a]

I. STARTING ASSUMPTIONS

The metric conditions considered in S.&Z.[1] were:

$$\begin{array}{ll} m_b = M_i = constant, & C_{zz} = 0 \\ m_b = M_f = constant, & C_{zz} \neq 0. \end{array} \tag{I.1}$$

Here, I will consider a particular scattering configuration where instead $C_{zz} = 0$ both initially and finally. Moreover, I will restrict myself to situations where the envelope of $C_{zz}(u)$ has a finite u integral at each point on the sphere. Under these conditions:

$$\begin{array}{ll} \int du u \partial_u C_{zz} & = u C_{zz}|_{-\infty}^{\infty} - \int du C_{zz} \\ & = -\int du C_{zz} \end{array} \tag{I.2}$$

where the boundary term can be dropped for quick enough $C_{zz}(u)$ fall-offs, which I will assume.

II. TWO DETECTOR PRIMER

Following S.&Z., I will consider detectors that are at a fixed $r = r_0$, with $\delta z = z' - z$ describing their angular separation in complex coordinates. If we assume δz is small, then:

$$L = \frac{2r_0|\delta z|}{1 + z\bar{z}} \tag{II.1}$$

is their spatial separation using the standard flat metric:

$$ds_F^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}. \tag{II.2}$$

When, in addition to the flat metric, there is a perturbation:

$$\begin{array}{ll} ds^2 = & ds_F^2 + \frac{2m_B}{r}du^2 \\ & + rC_{zz}dz^2 + D^zC_{zz}dudz + c.c., \end{array} \tag{II.3}$$

the trajectories of light rays traveling between detectors will satisfy:

$$2r_0^2\gamma_{z\bar{z}}\delta z\delta\bar{z} + r_0C_{zz}\delta z^2 + D^zC_{zz}\delta_{zz'}u\delta z + c.c. - (\delta_{zz'}u)^2 = 0 \tag{II.4}$$

going from $z \to z'$, whereas the reverse route will have:

$$2r_0^2\gamma_{z\bar{z}}\delta z\delta\bar{z} + r_0C_{zz}\delta z^2 - D^zC_{zz}\delta_{z'z}u\delta z + c.c. - (\delta_{z'z}u)^2 = 0 \tag{II.5}$$

where the Bondi mass term is subleading in r_0. Subtracting the two equations gives:

$$\delta_{zz'}u - \delta_{z'z}u = D^zC_{zz}\delta z + c.c. \tag{II.6}$$

While adding the two equations gives:

$$(\delta_{zz'}u)^2 + (\delta_{z'z}u)^2 = 4r_0^2\gamma_{z\bar{z}}\delta z\delta\bar{z} + r_0C_{zz}\delta z^2 + c.c. \tag{II.7}$$

Combing these yields:

$$\delta_{zz'}u = \tilde{L} + \frac{1}{2}[D^zC_{zz}\delta z + c.c.] \tag{II.8}$$

$$\delta_{z'z}u = \tilde{L} - \frac{1}{2}[D^zC_{zz}\delta z + c.c.] \tag{II.9}$$

where

$$\tilde{L} = L + \frac{r_0}{2L}[C_{zz}\delta z^2 + c.c.] \tag{II.10}$$

III. SUBLEADING MEASUREMENT

Consider N detectors arranged in a regular polygon around z=0.

$$z_n = \epsilon e^{i\frac{2\pi n}{N}}, \quad n \in \{0, ..., N-1\} \tag{III.1}$$

In the large N limit, one has:

$$z = \epsilon e^{i\phi}, \quad \delta z = iz\delta\phi \tag{III.2}$$

The difference between a clockwise versus a counter clockwise circuit for a constant C_{zz} is:

$$\begin{array}{l} \lim\limits_{N\to\infty} \sum\limits_{n=0}^{N-1} \{\delta_{n,n+1}u - \delta_{n+1,n}u\} \\ = \lim\limits_{N\to\infty} \sum\limits_{n=0}^{N-1} D^zC_{zz}(\epsilon e^{i\frac{2\pi n}{N}}) i\epsilon e^{i\frac{2\pi n}{N}} \frac{2\pi}{N} + c.c. \\ = \int\limits_0^{2\pi} D^zC_{zz}(z) iz\delta\phi + c.c. \\ = \oint_\epsilon D^zC_{zz}dz + c.c. \end{array} \tag{III.3}$$

This is equivalent to sending the signal chains in opposite directions around the array of detectors and looking at the time difference when the two pulses arrive at the starting point after a single loop.

Note that this is a cumulative effect. The difference in timing is a correction to the net time for a single circuit, which is at leading order in r_0 is:

$$L_\epsilon = 4\pi r_0\epsilon. \tag{III.4}$$

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[a] ISBN: 978-0-9863685-9-2

[1] arXiv:1411.5745v1.pdf

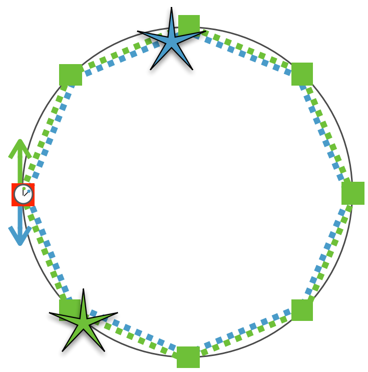

FIG. 1. Two counter propagating relays of signals are triggered sequentially around the circular array of detectors, with the final de-sychronization recorded.

The subleading correction, which does not change the relative time delay of the counter rotating signals, is due to the difference between the loop integrals of L and $\tilde{L}$.

Consider a $C_{zz}(u)$ which varies slowly enough that summing the accumulated time delay for intervals spaced by $\Delta u = 4\pi r_0 \epsilon$ approximates the u integral:

$$\begin{aligned} &\sum_m \oint_\epsilon D^z C_{zz}(u = 4\pi r_0 \epsilon m) dz + c.c. \\ &= \tfrac{1}{4\pi r_0 \epsilon} \int du \oint_\epsilon D^z C_{zz} dz + c.c. \end{aligned} \tag{III.5}$$

Under the assumption that the u integral is finite, the final time delay between two initially synched counter rotating signal chains thus corresponds to u, z integral.

IV. SOFT FACTOR INTERPRETATION

The net de-synchronization is related to the subleading soft graviton factor. For a stationary metric satisfying the the vacuum Einstein equations, it would be the time integral of the curl of the twist, which would be zero. In a scattering process with gravitational radiation, however, one can use the expectation value interpretation to think of $\int du D_{\bar{z}}^2 C_{zz}$ in terms of the soft factor. In this case, the real and imaginary parts of the subleading soft factor correspond to divergence and curl of $D^a C_{ab}$. Note that the subleading soft graviton theorem contains more content than that used for the superrotation charge. The real part of $D_{\bar{z}}^2 C_{zz}$ is what appears in the superrotation charge, $\int du D^a D^b C_{ab}$, while the imaginary part contributes to $i \int du [D_{\bar{z}}^2 C_{zz} - D_z^2 C_{\bar{z}\bar{z}}]$. This imaginary part is expected to be finite under the boundary conditions[2] $D_z^2 C_{\bar{z}\bar{z}} - D_{\bar{z}}^2 C_{zz} = 0$ at $\mathcal{I}_+^+$ and $\mathcal{I}_-^+$, assuming quick enough fall offs in u for this quantity.

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[2] arXiv:1406.3312.pdf