Flat space amplitudes in a highest-weight basis:

A natural choice for studying implications of superrotations?

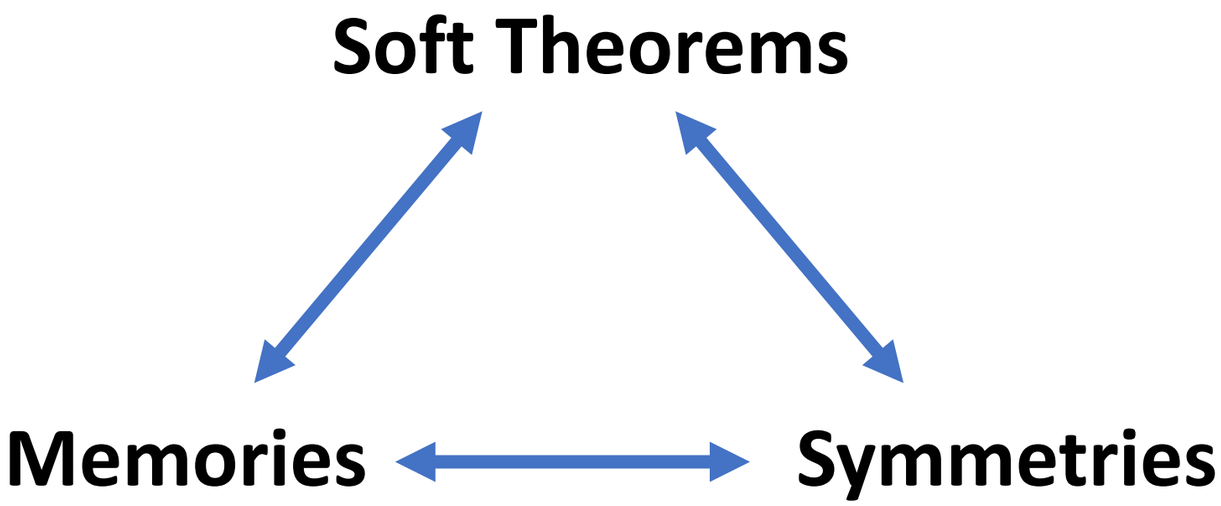
SABRINA GONZALEZ PASTERSKI

A Triangle of Relations

- Over the past few years Strominger and collaborators have been studying how asymptotic symmetry groups manifest themselves in scattering matrix elements via soft factors
  - **soft factors** ⇒ **Ward identities** implying constraints on the $\mathcal{S}$ -matrix

    which correspond to a **larger class** of symmetry transformations

- What resulted is a pattern of connections between traits of low energy radiation that appeared with multiple iterations: this turns into a fill in the blank exercise once one vertex of a new iteration is motivated



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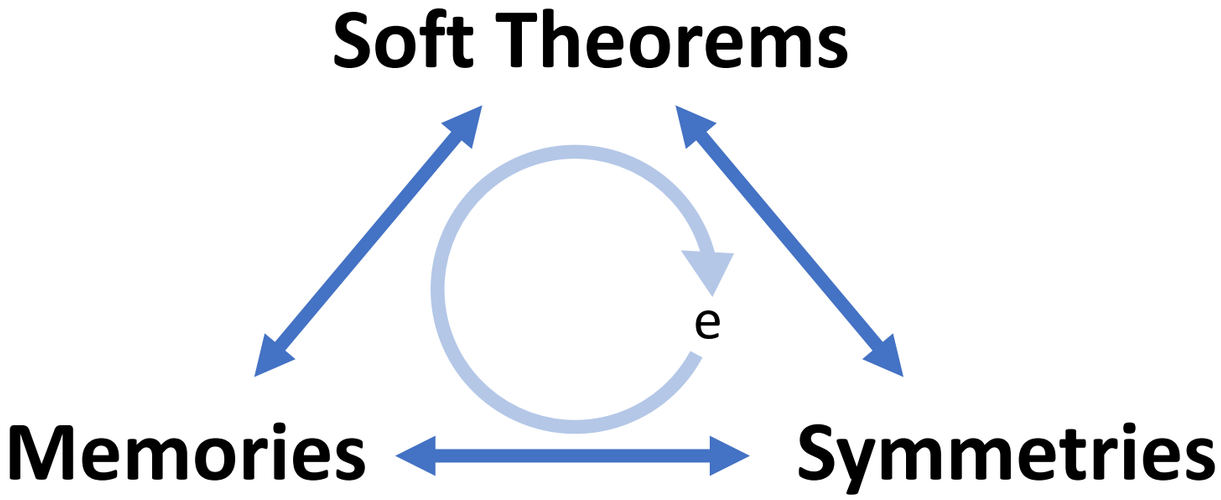

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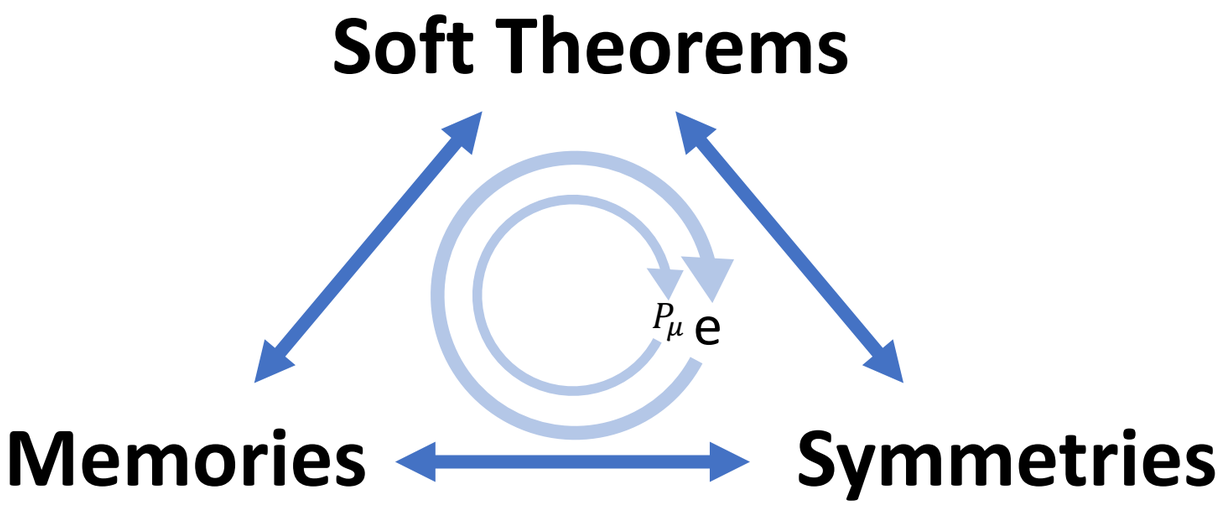

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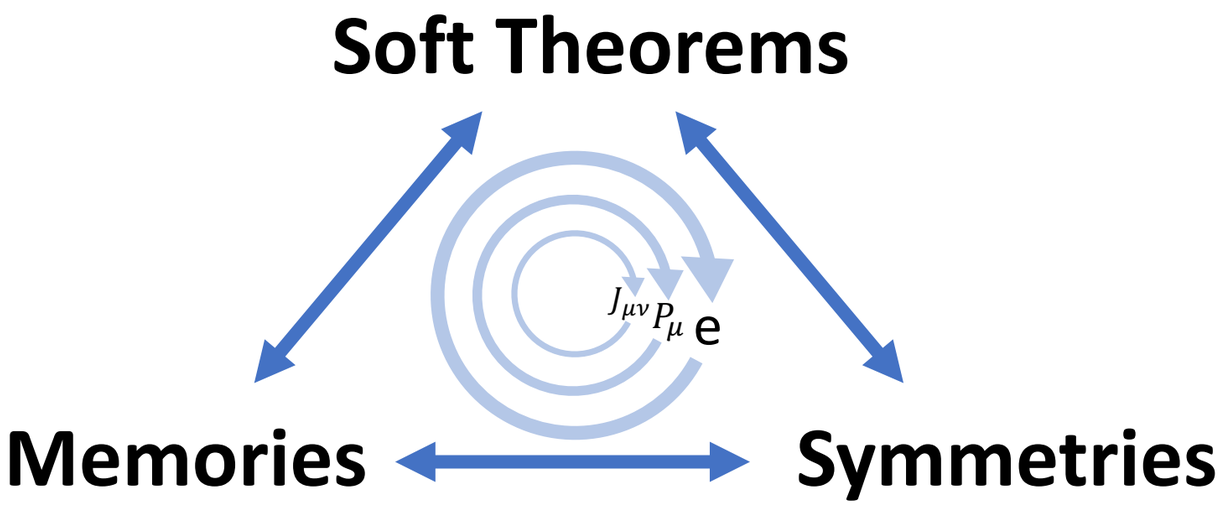

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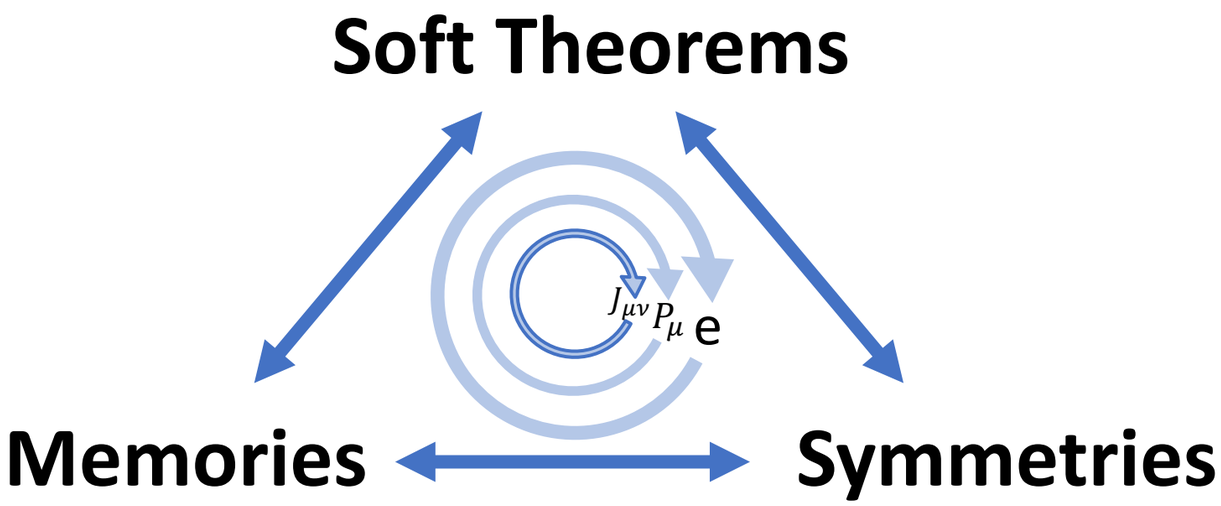
- What resulted is a pattern of connections between traits of low energy radiation that appeared with multiple iterations: this turns into a fill in the blank exercise once one vertex of a new iteration is motivated

In this manner a brand new iteration was completed (ASG ~ past decade vs 50's-60's). This iteration is related to a generalization of Lorentz transformations and has motivated looking at $\mathcal{S}$-matrix elements in a new basis with definite SL(2,C) weights



A Triangle of Relations

Goal for this talk:
1) explain the vertices of this soft triangle to demonstrate the most recent spin-related iteration
2) how this motivates a change of basis (current work with S.H. Shao and A. Strominger)

i) Weinberg – photon O($\frac{1}{\omega}$)
ii) Weinberg – graviton O($\frac{1}{\omega}$)
iii) Cachazo & Strominger – graviton O(1)

Soft Theorems

Memories

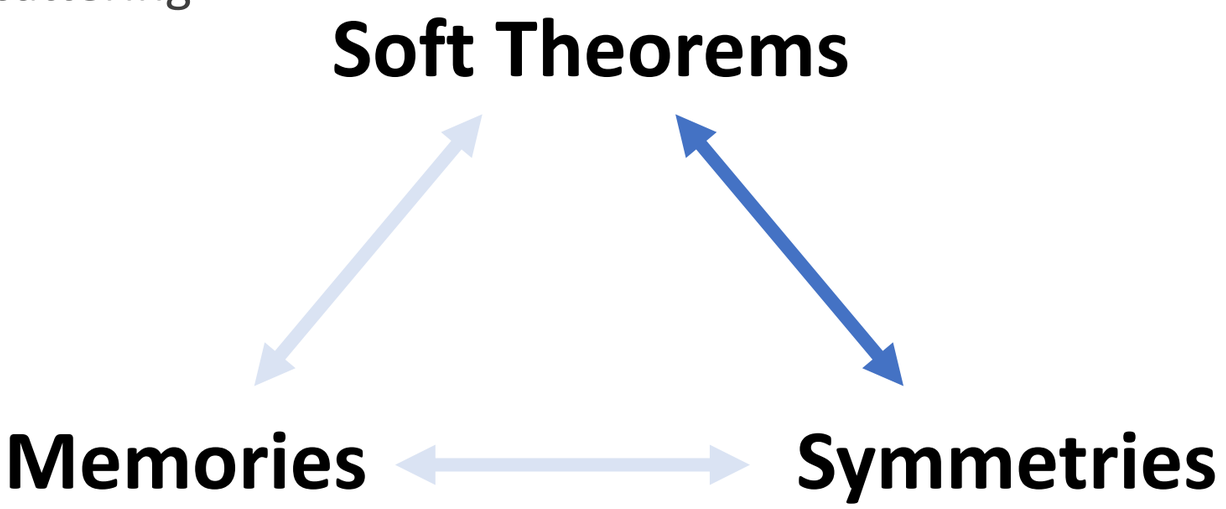
i) Liénard-Wiechert / Bieri & Garfinkle
ii) Zeldovich & Polnarev / Christodoulou
iii) Pasterski, Strominger, & Zhiboedov

Symmetries

(global)	(asymptotic)
i) e-charge	large U(1)
ii) P_μ	supertranslations
iii) $\boldsymbol{J_{\mu\nu}}$	**superrotations**

$\mathcal{S}$-matrix Constraints from Symmetries

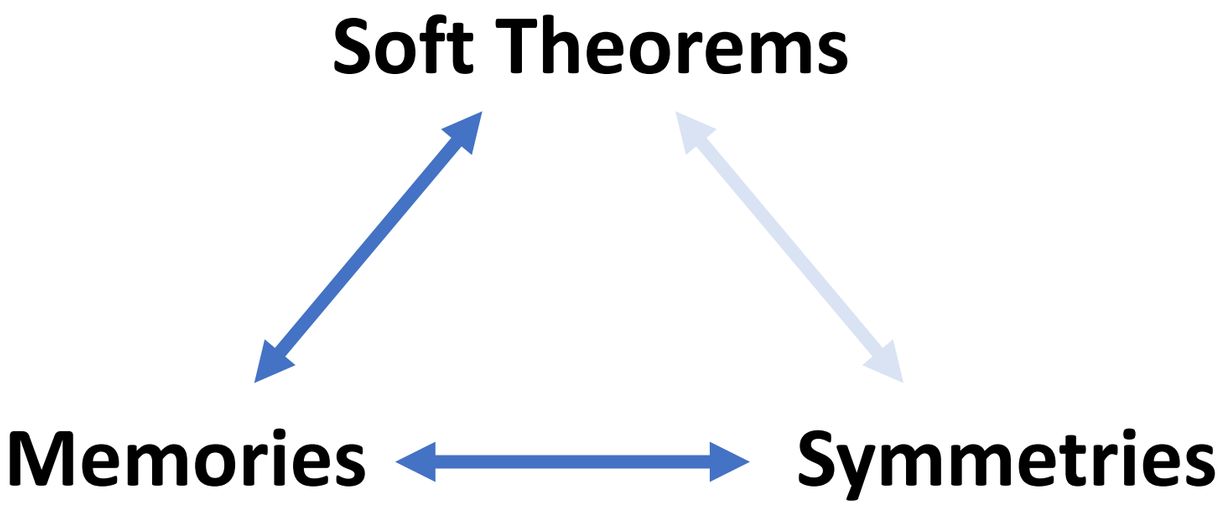
- Noether's Theorem: Continuous Symmetries ⇒ Conservation Laws
- More Symmetries ⇒ More Constraints on $\mathcal{S}$-matrix
- Modus Operandi:
  - Look for larger set of "physical" symmetries
  - Motivate via properties of low energy scattering



$\mathcal{S}$-matrix Constraints from Symmetries

Where do these "extra" symmetries come from?

- Boundary conditions can't be too restrictive so as to disallow typical scattering processes, larger class of boundary conditions gives a larger group of symmetries that preserve this class of boundary conditions
  - These are our extra symmetries



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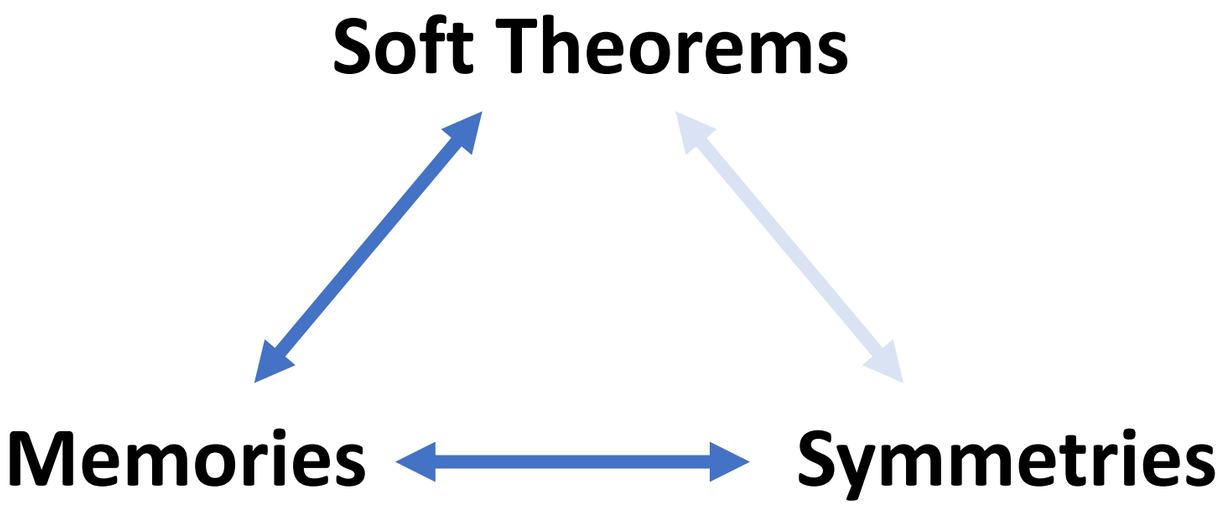
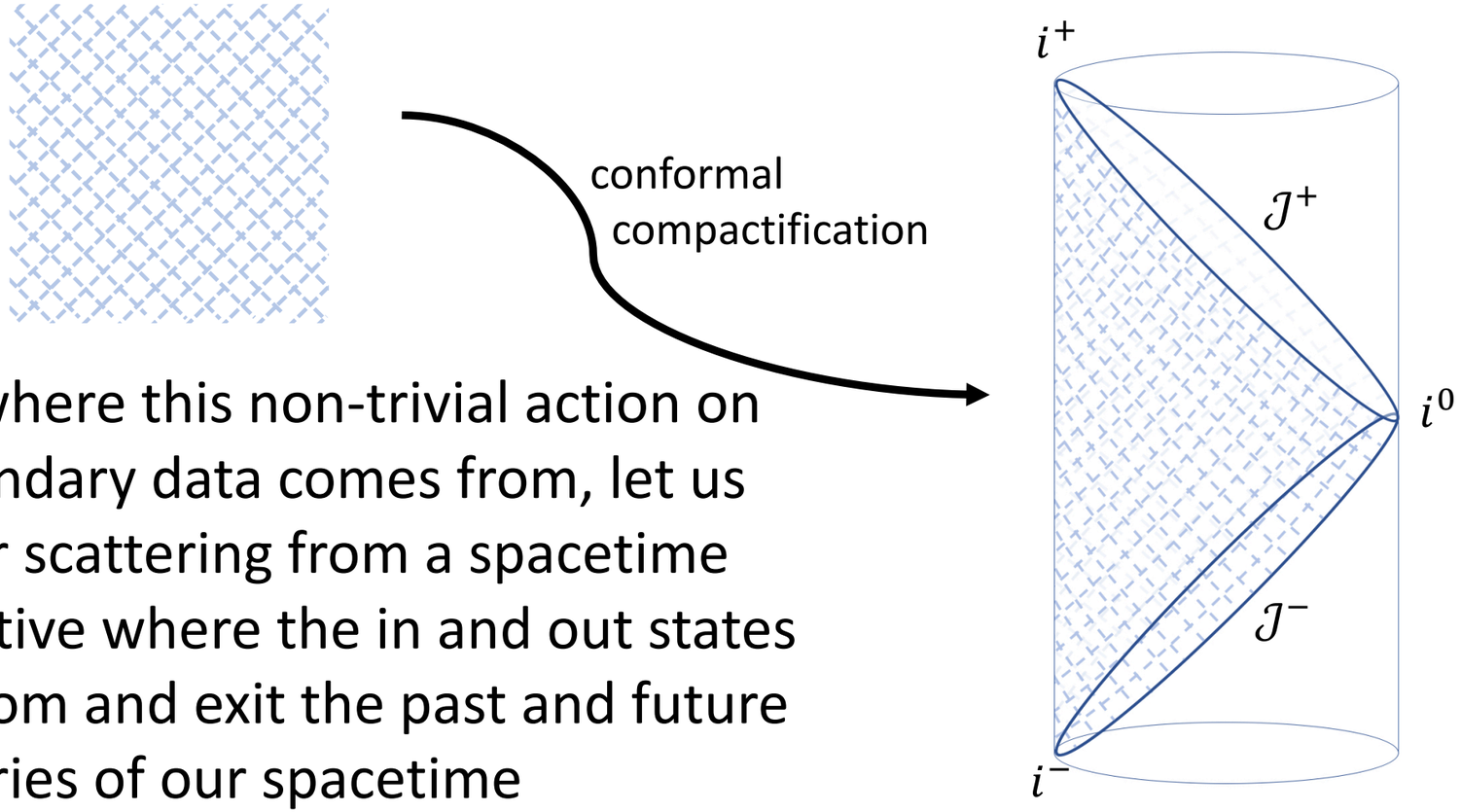
Class of gauge transformations that act non-trivially on the boundary data which is larger than the standard global U(1) of E&M or P_μ and $J_{\mu\nu}$ of Minkowski space

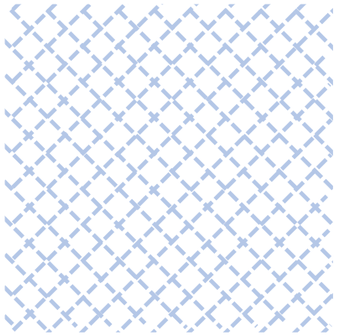


Scattering from a Spacetime Perspective

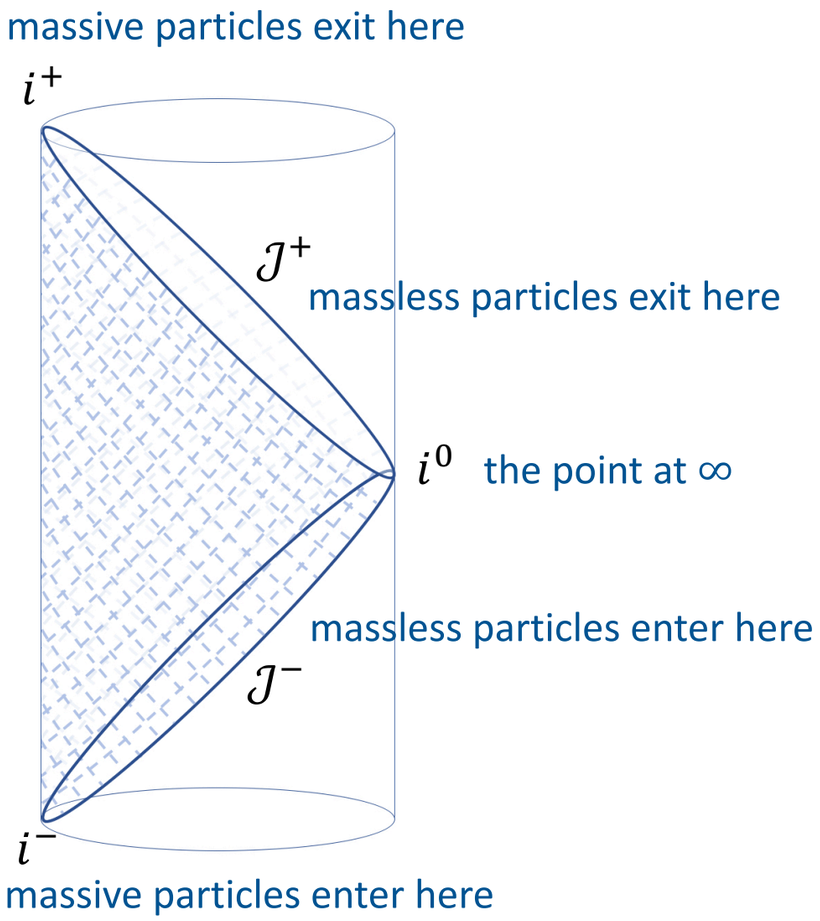


To see where this non-trivial action on the boundary data comes from, let us consider scattering from a spacetime perspective where the in and out states come from and exit the past and future boundaries of our spacetime

Scattering from a Spacetime Perspective



conformal compactification



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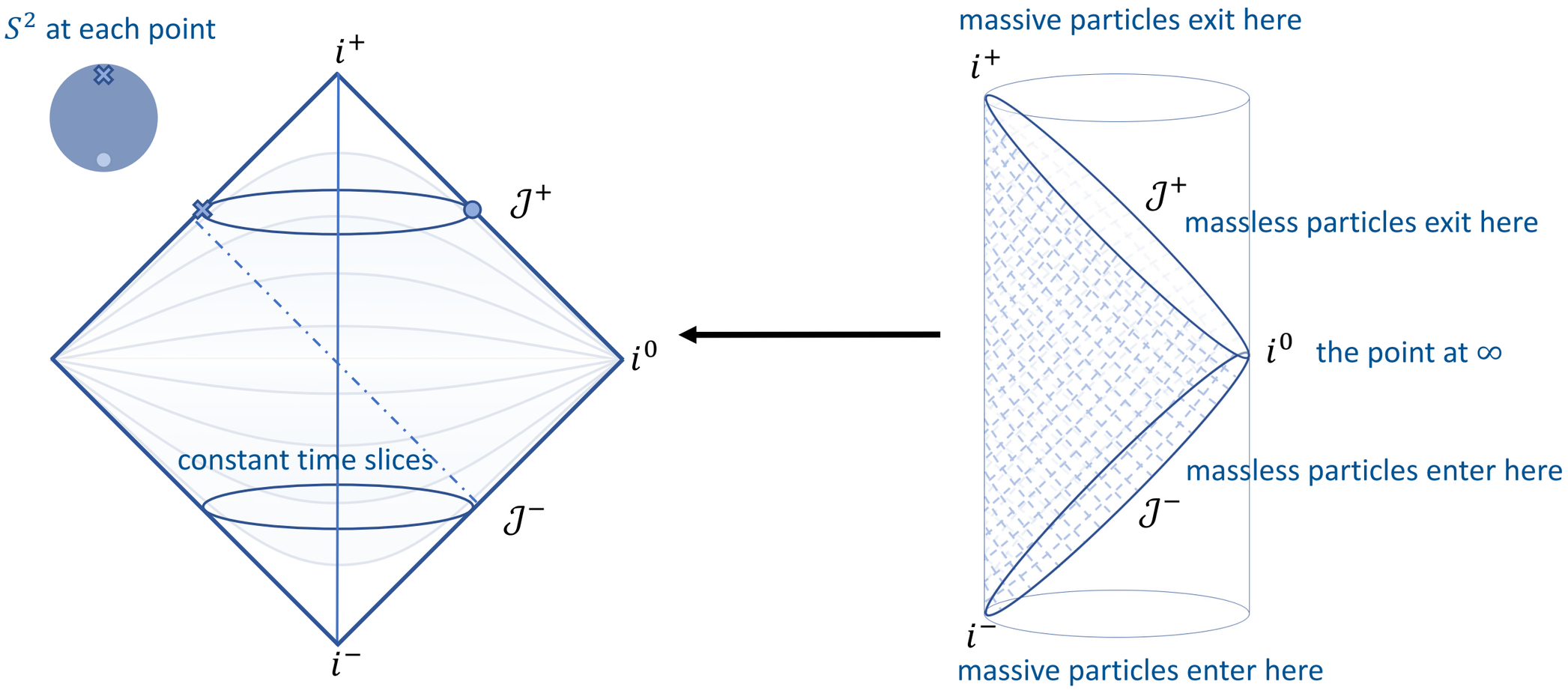
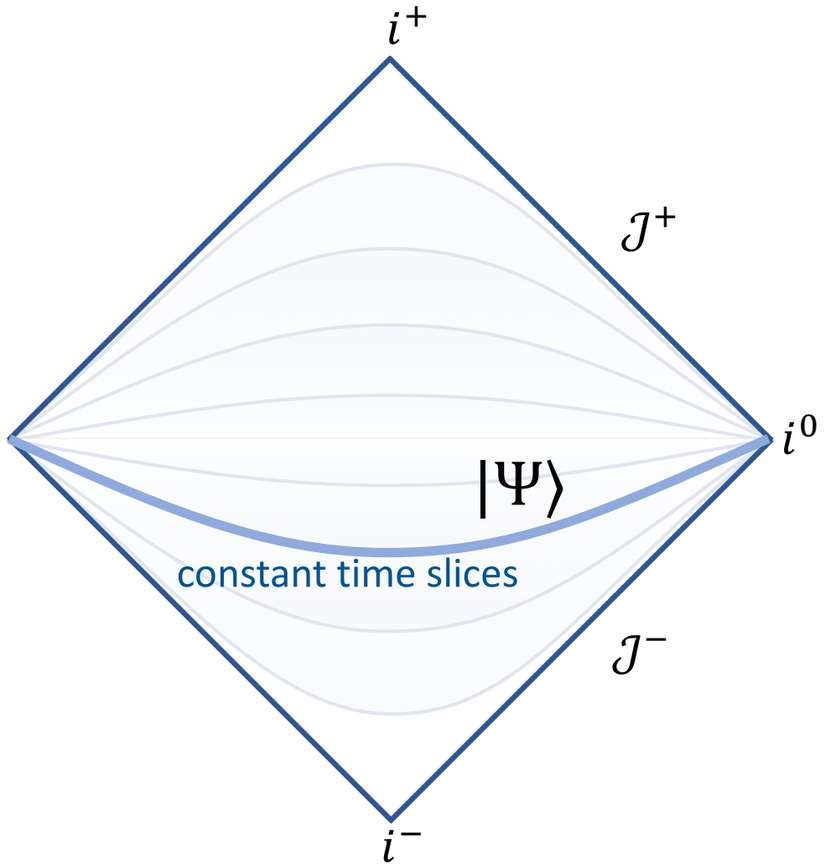
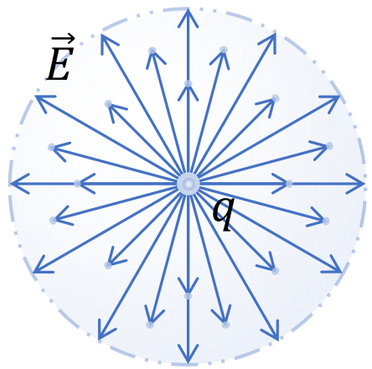


U(1) Example

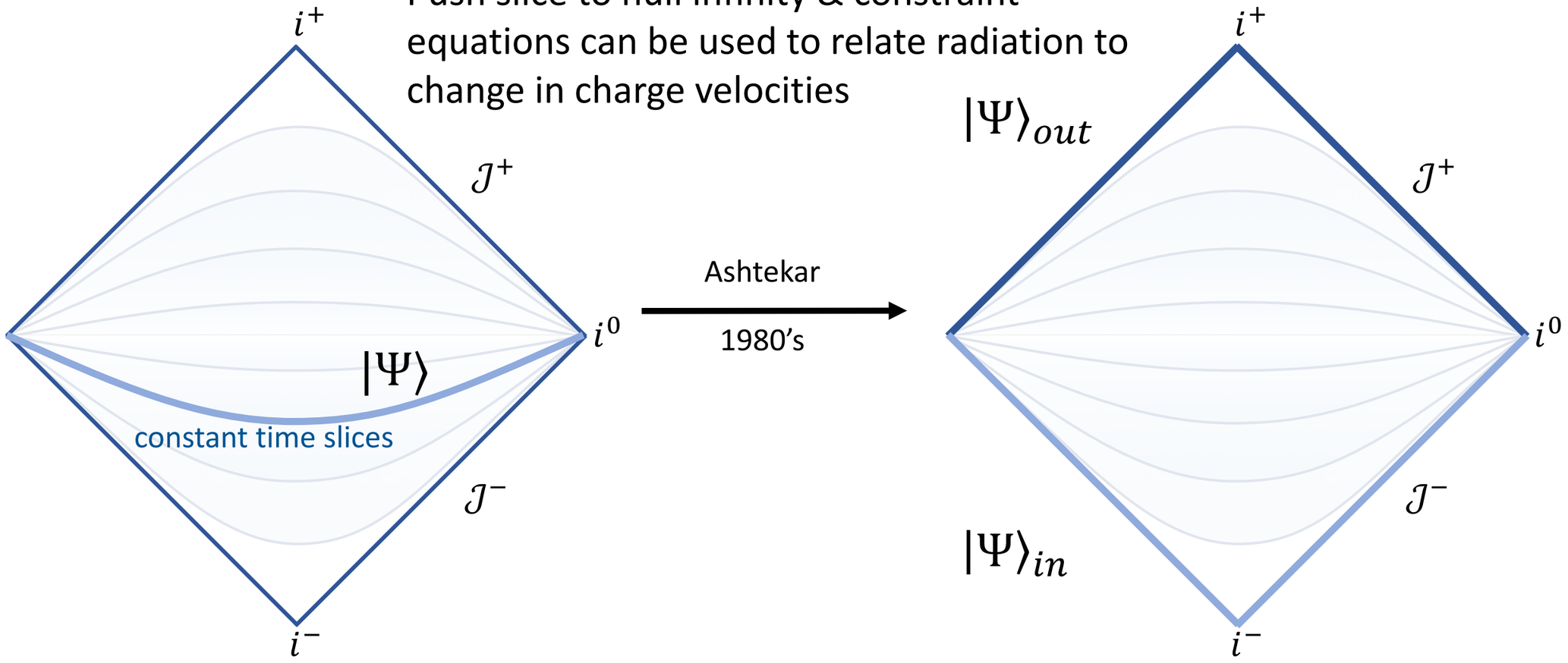


- With this in mind we can understand the U(1) iteration of the triangle
- Gauss's law is a constraint equation relating electric flux to charges



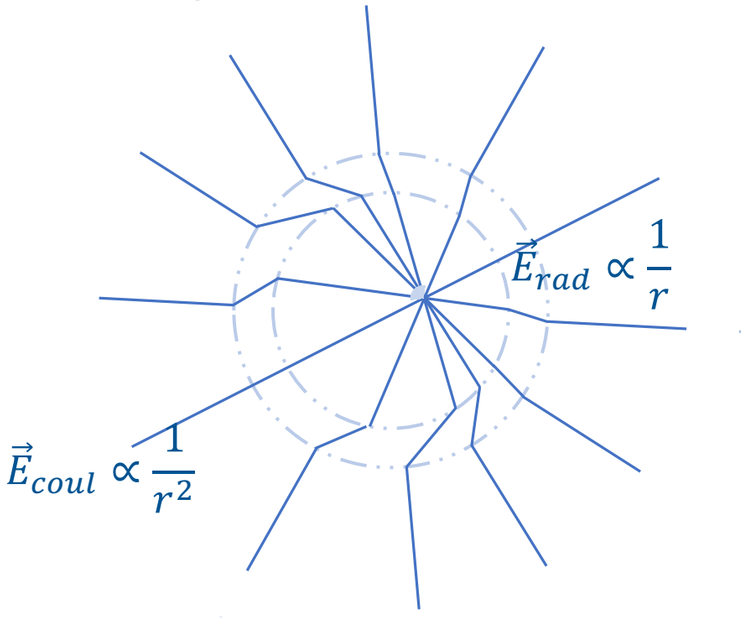
U(1) Example

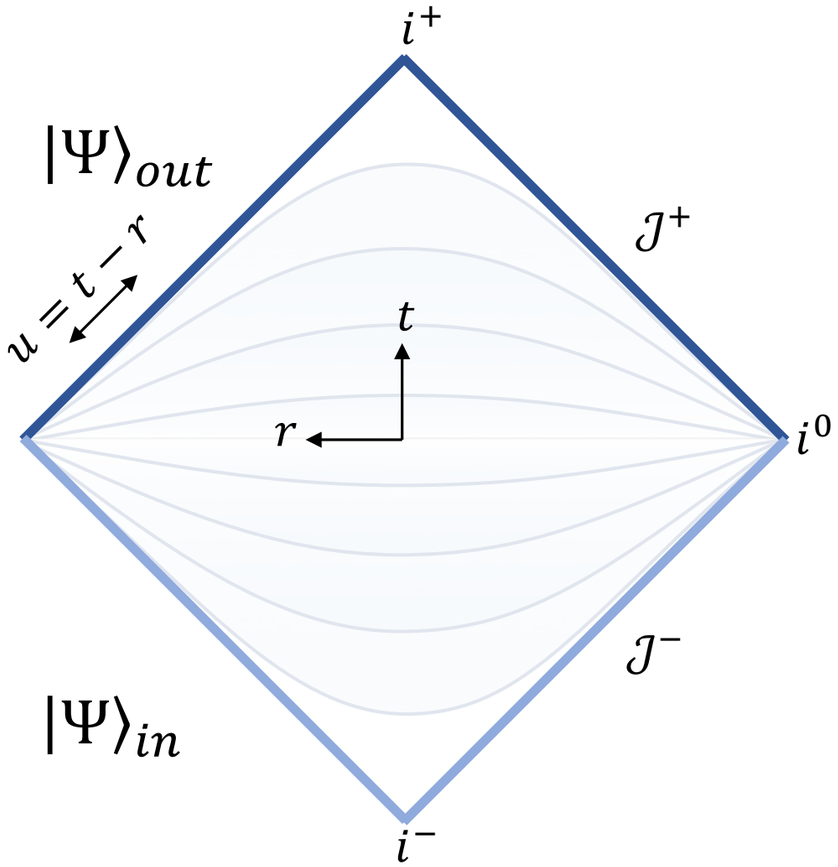
Push slice to null infinity & constraint equations can be used to relate radiation to change in charge velocities



U(1) Example

- Look at radial fall offs and isolate the free data corresponding to radiative modes
- Find the ASG that preserve these fall-offs





U(1) Example

Some more details:

- Radial Expansion:

$$\mathcal{A}_z(r,u,z,\bar{z}) = A_z(u,z,\bar{z}) + \sum_{n=1}^{\infty} \frac{A_z^{(n)}(u,z,\bar{z})}{r^n}$$

$$\mathcal{A}_u(r,u,z,\bar{z}) = \frac{1}{r}A_u(u,z,\bar{z}) + \sum_{n=1}^{\infty} \frac{A_u^{(n)}(u,z,\bar{z})}{r^{n+1}}$$

$$F_{ur} = A_u$$
$$F_{z\bar{z}} = \partial_z A_{\bar{z}} - \partial_{\bar{z}} A_z$$
$$F_{uz} = \partial_u A_z$$

- ASG that preserves this expansion:

$$\delta_\epsilon A_z(u,z,\bar{z}) = \partial_z \epsilon(z,\bar{z})$$

- Mode Expansion:

$$\mathcal{A}_\mu(x) = e\sum_{\alpha=\pm}\int \frac{d^3q}{(2\pi)^3}\frac{1}{2\omega_q}\left[\epsilon_\mu^{\alpha^*}(\vec{q})a_\alpha(\vec{q})e^{iq\cdot x} + \epsilon_\mu^{\alpha}(\vec{q})a_\alpha(\vec{q})^\dagger e^{-iq\cdot x}\right]$$

- Constraint Equation:

$$\partial_u A_u = \partial_u(D^z A_z + D^{\bar{z}} A_{\bar{z}}) + e^2 j_u$$

[arXiv:1407.3789]

Coordinate Conventions:

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$$

$$z = e^{i\phi}\tan\frac{\theta}{2} \quad \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$

U(1) Example

Two key points:

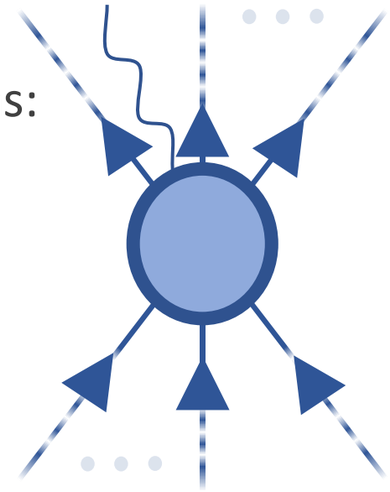
- Saddle point at large r picks out a gauge boson momentum pointing in the same direction as where an observer near null infinity would detect it. As a result, one ends up with a mode expansion where the angular integral localizes, and (u, ω) remain as Fourier conjugates.

$$e^{iq\cdot x} = e^{-i\omega u - i\omega r(1-\hat{q}\cdot\hat{x})} \quad \Rightarrow \quad A_z(u,z,\bar{z}) = -\frac{i}{8\pi^2}\frac{\sqrt{2}e}{1+z\bar{z}}\int_0^\infty d\omega\left[a_+(\omega\hat{x})e^{-i\omega u} - a_-(\omega\hat{x})^\dagger e^{i\omega u}\right]$$

- $\int du$ picks out $\omega \to 0$. As such we can relate the soft factors to the constraint equations: Fourier transform of a pole $\frac{1}{\omega}$ is a step function

$$S^{(0)-} = \sum_k eQ_k \frac{p_k\cdot\epsilon^-}{p_k\cdot q}$$

$$\langle z_{n+1}, z_{n+2}, ...|a_-(q)\mathcal{S}|z_1, z_2, ...\rangle = S^{(0)-}\langle z_{n+1}, z_{n+2}, ...|\mathcal{S}|z_1, z_2, ...\rangle + \mathcal{O}(1)$$

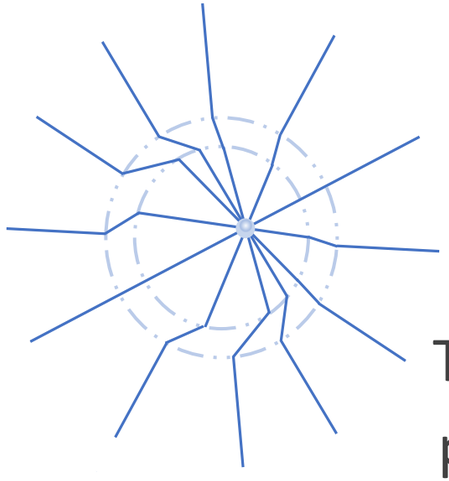


[arXiv:1407.3789, arXiv:1505.00716]

U(1) Example

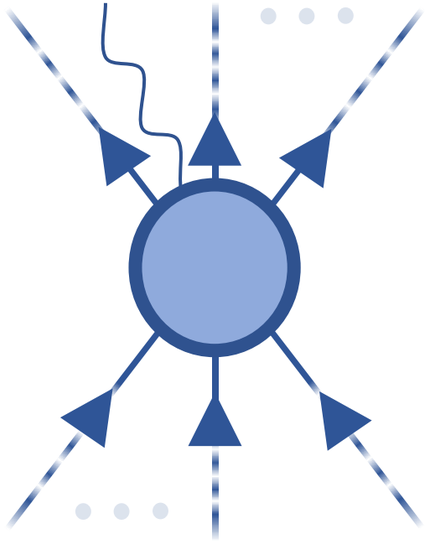
Integrate the constraint equation along u

$$E_r = \frac{Q}{4\pi r^2}\frac{1}{\gamma^2(1-\vec{\beta}\cdot\hat{n})^2}$$



$$\Delta A_u = 2D^z \Delta A_z + e^2 \int du j_u$$

$$-\frac{e}{4\pi}\lim_{\omega\to 0}\omega[D^z\hat{\epsilon}_z^{*+}S_p^{(0)+} + D^{\bar{z}}\hat{\epsilon}_{\bar{z}}^{*-}S_p^{(0)-}] \quad = -e^2\frac{Q}{4\pi}\frac{1}{\gamma^2(1-\vec{\beta}\cdot\hat{n})^2}$$



The soft factor indicates that typical scattering processes will produce a nonzero u integrated electric field.

Some Conventions:

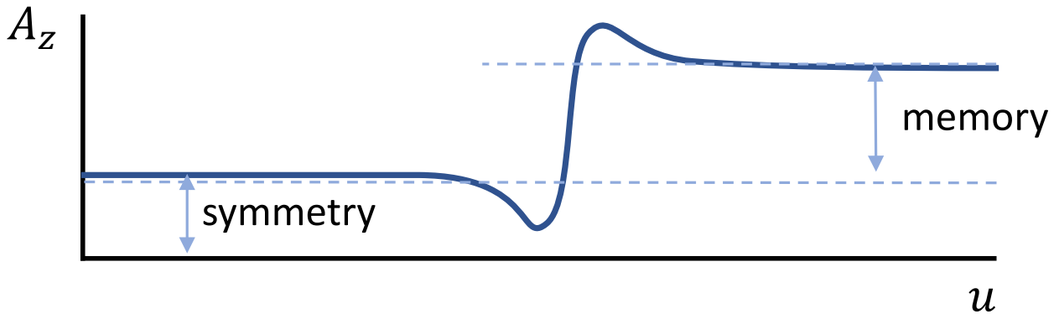
$$p^\mu = m\gamma(1,\vec{\beta}) \qquad S_p^{(0)\pm} = eQ\frac{p\cdot\epsilon^\pm}{p\cdot q}$$

$$\Delta A_z = -\frac{e}{4\pi}\hat{\epsilon}_z^{*+}\omega S^{(0)+}$$

[arXiv:1505.00716]

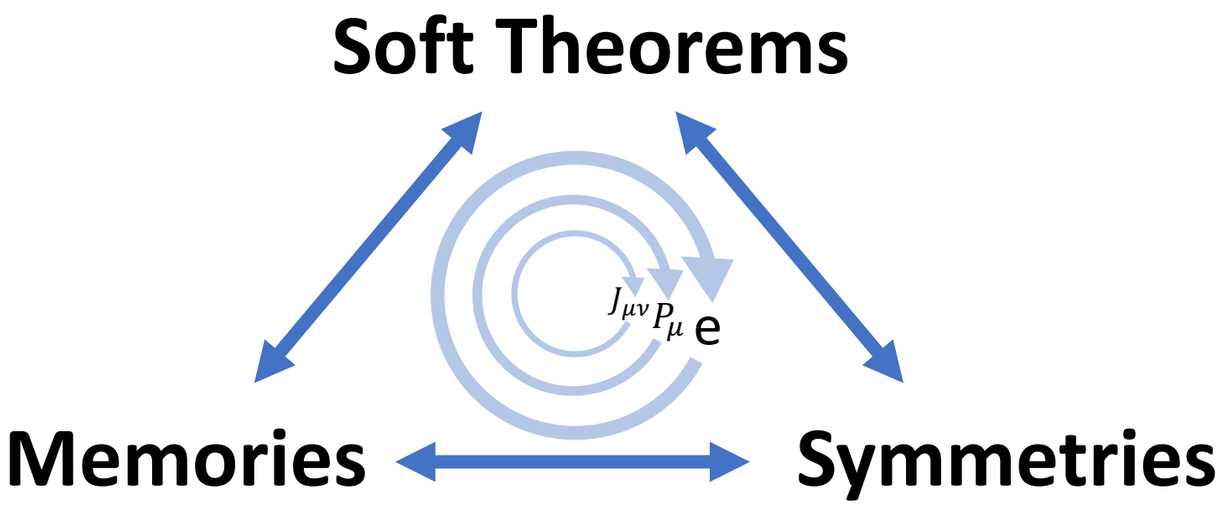
U(1) Example

- Upshot: The residue of the Weinberg pole indicates a nonzero value for certain low-energy radiation observables aka **"memory effects"**
- Since setting these modes to zero would trivialize the allowed scattering events, we get with this class of boundary conditions a larger class of gauge transformations that preserve the radial order of the falloffs while shifting the boundary values aka **"large gauge transformations"** (see Strominger [arXiv:1308.0589 arXiv:1312.2229] for how soft theorems can be used to construct Ward identities for these asymptotic symmetries)



Midpoint Recap

- From the U(1) example we've seen how from soft theorems we can extract low energy radiation observables (memory effects), which are non-zero in typical scattering processes and thus obtain a larger asymptotic symmetry group for less stringent boundary conditions
- For the gravity case, we can write down a metric expansion, holding on to the notion of null infinity for spacetimes that are only asymptotically flat, look at the vector fields for diffeomorphisms which preserve these falloffs



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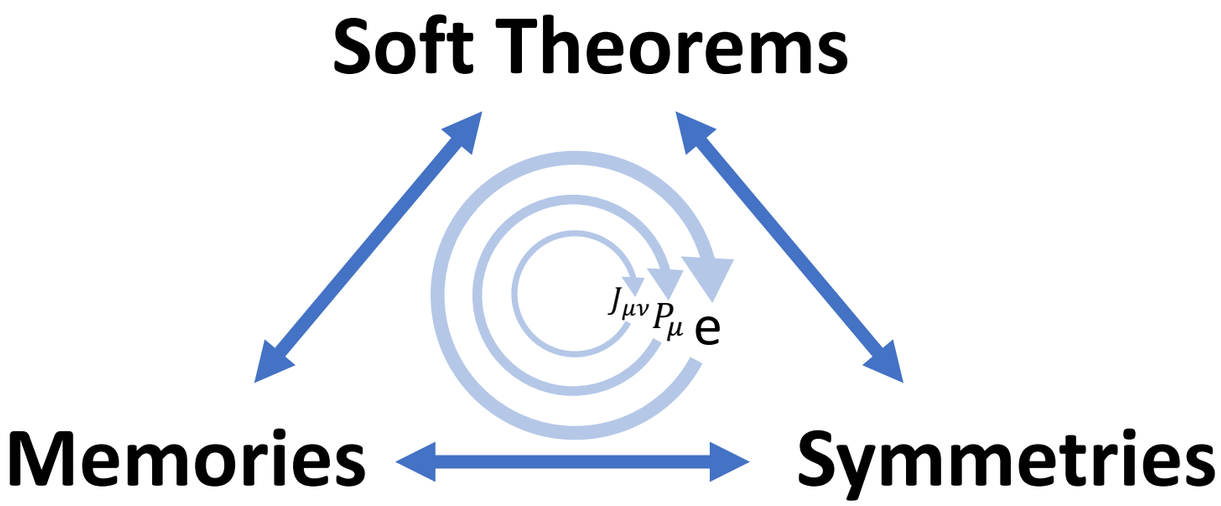
- For the gravity case, we can write down a metric expansion, holding on to the notion of null infinity for spacetimes that are only asymptotically flat, look at the vector fields for diffeomorphisms which preserve these falloffs

Both the subleading soft factor and the ASG it corresponds to will motivate looking at $\mathcal{S}$ -matrix elements in a new basis with definite SL(2,C) weights, while the memory effect helps motivate this extended-BMS symmetry as "physical."



Asymptotically Flat Spacetimes

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

Want to consider non-trivial gravitational backgrounds that are “close” to being flat

BMS 1960’s

- Approach flat spacetime far away from sources

Asymptotically Flat Spacetimes

Some more details:

- Radial Expansion:

$$\begin{aligned} ds^2 &= -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} + 2\frac{m_B}{r}du^2 \\ &+\left(rC_{zz}dz^2 + D^zC_{zz}dudz + \tfrac{1}{r}(\tfrac{4}{3}N_z - \tfrac{1}{4}\partial_z(C_{zz}C^{zz}))dudz + c.c.\right) + \ldots \end{aligned}$$

- ASG that preserves this expansion:

$$\begin{aligned} \xi^+ =&(1+\frac{u}{2r})Y^{+z}\partial_z - \frac{u}{2r}D^{\bar{z}}D_zY^{+z}\partial_{\bar{z}} - \frac{1}{2}(u+r)D_zY^{+z}\partial_r + \frac{u}{2}D_zY^{+z}\partial_u + c.c \\ &+ f^+\partial_u - \frac{1}{r}(D^zf^+\partial_z + D^{\bar{z}}f^+\partial_{\bar{z}}) + D^zD_zf^+\partial_r \end{aligned}$$

- Mode Expansion: C_{zz} radiative data

- Constraint Equations: $G_{uu} = 8\pi GT^M_{uu} \quad G_{uz} = 8\pi GT^M_{uz}$

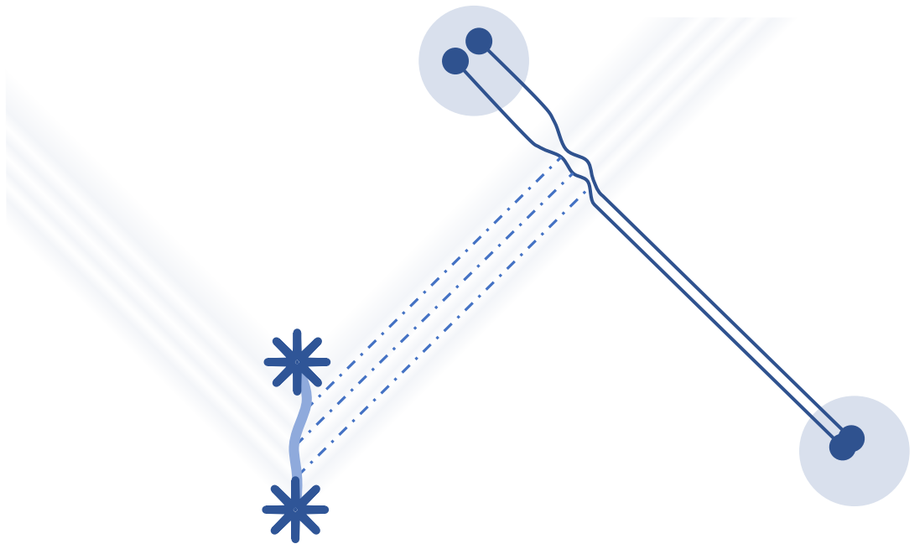
Coordinate Conventions:

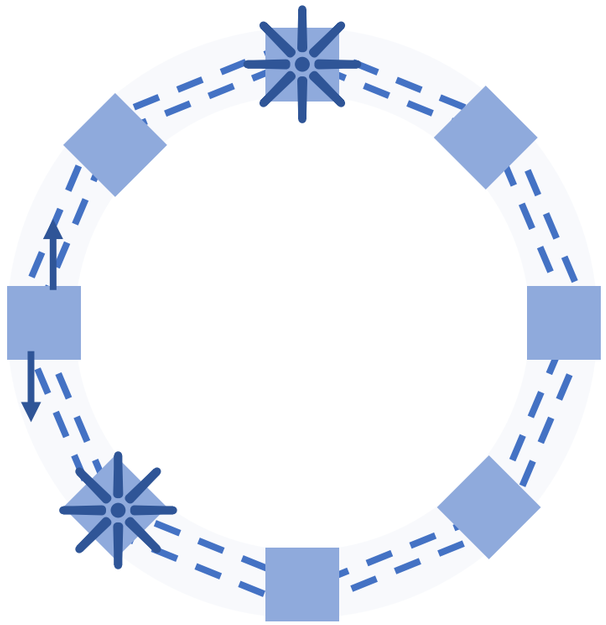
$$z = e^{i\phi}\tan\frac{\theta}{2} \qquad \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$

$$f^+ = f^+(z,\bar{z}) \qquad \partial_{\bar{z}}Y^{+z} = 0$$

Asymptotically Flat Spacetimes

The physical observable modes conjugate to the ASG transformations are gravitational displacement and **spin memory** effects ([**arXiv:1502.06120**], potentially observable [**arXiv:1702.03300**])

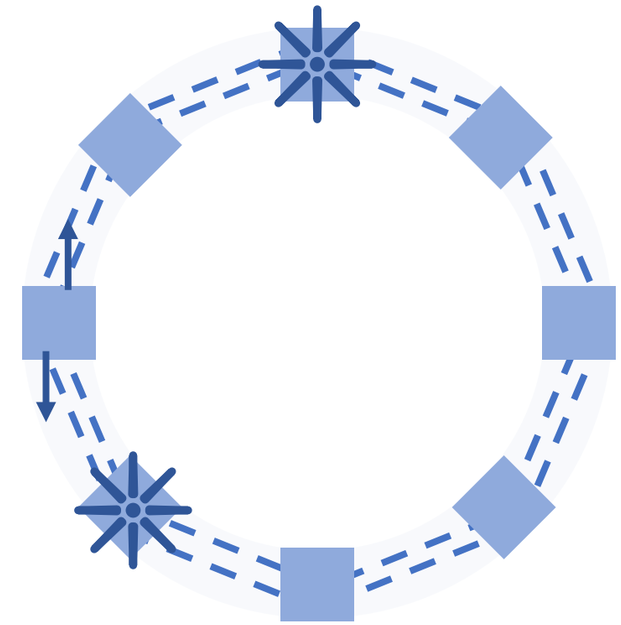




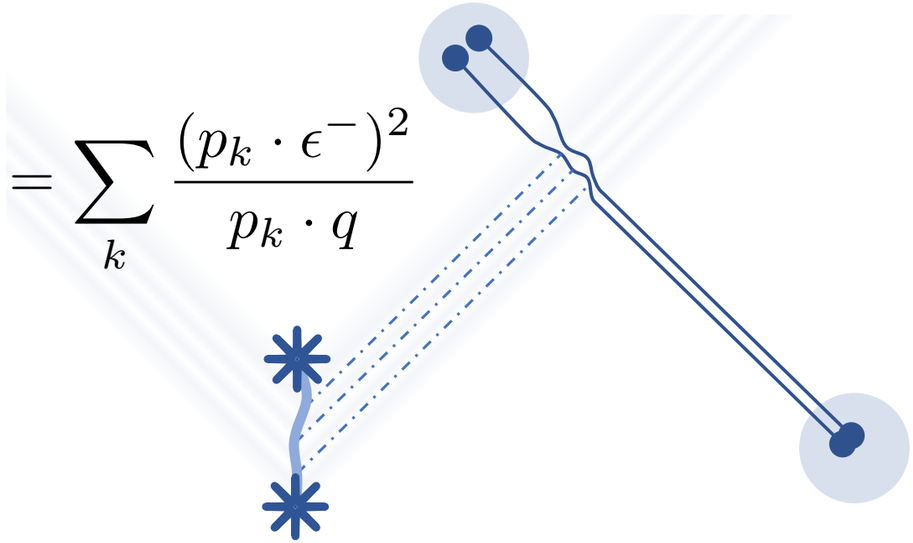
Asymptotically Flat Spacetimes

Their values can be extracted from the leading as well as a more recent subleading ([arXiv:1404.4091]) soft graviton theorems.

$$\langle z_{n+1}, z_{n+2}, ...|a_-(q)\mathcal{S}|z_1, z_2, ...\rangle = \left(S^{(0)-} + S^{(1)-}\right)\langle z_{n+1}, z_{n+2}, ...|\mathcal{S}|z_1, z_2, ...\rangle + \mathcal{O}(\omega)$$



$$S^{(0)-} = \sum_k \frac{(p_k \cdot \epsilon^-)^2}{p_k \cdot q}$$



$$S^{(1)-} = -i \sum_k \frac{p_{k\mu}\epsilon^{-\mu\nu} q^\lambda J_{k\lambda\nu}}{p_k \cdot q}$$

Asymptotically Flat Spacetimes

- Let us take a closer look at the superrotation vector field near null infinity:

$$\xi^+|_{\mathcal{J}^+} = Y^{+z}\partial_z + \frac{u}{2}D_z Y^{+z}\partial_u + c.c.$$

  - Notice with two copies of Witt algebra since Y is any 2D CKV
  - Also, $u\partial_u$ prefers Rindler energy eigenstates

Weight Conventions:

$$h = \frac{1}{2}(s+1+iE_R) \quad \bar{h} = \frac{1}{2}(-s+1+iE_R)$$

$$\Delta = h + \bar{h} \quad s = h - \bar{h}$$

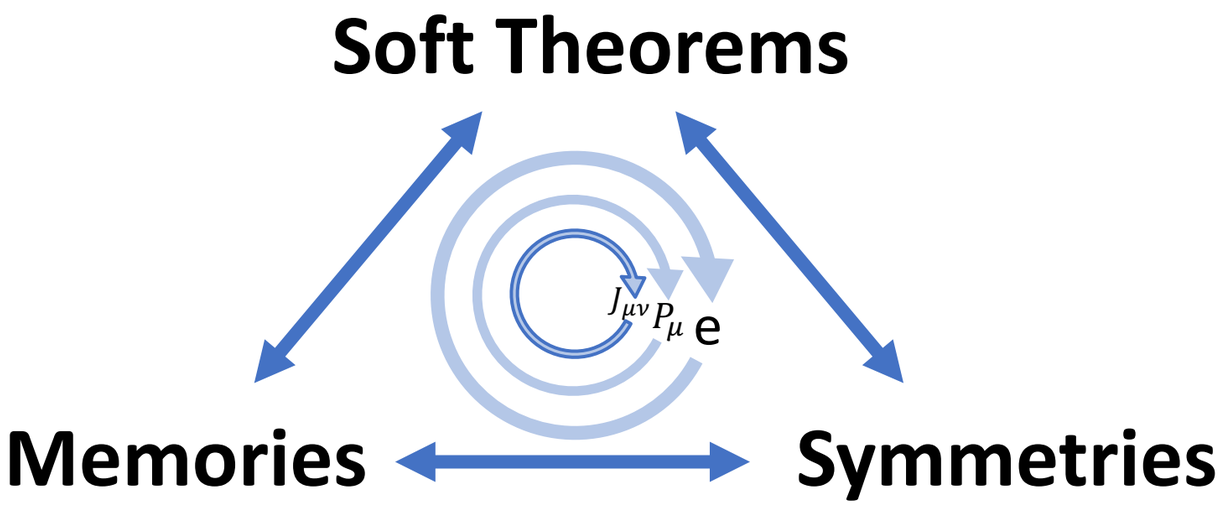
- Rather than using the subleading soft factor to establish a Ward identity for this asymptotic symmetry ([arXiv:1406.3312]) one can massage it to look like 2D stress tensor ([arXiv:1609.00282])

$$T_{zz} \equiv \frac{i}{8\pi G}\int d^2w \frac{1}{z-w} D_w^2 D^{\bar{w}} \int du u\partial_u C_{\bar{w}\bar{w}}$$

$$\langle T_{zz}\mathcal{O}_1\cdots\mathcal{O}_n\rangle = \sum_{k=1}^{n}\left[\frac{h_k}{(z-z_k)^2} + \frac{\Gamma^{z_k}_{z_k z_k}}{z-z_k}h_k + \frac{1}{z-z_k}\left(\partial_{z_k} - |s_k|\Omega_{z_k}\right)\right]\langle\mathcal{O}_1\cdots\mathcal{O}_n\rangle$$

Final Recap

- We see that a triangle of connections between soft theorems, memory effects, and asymptotic symmetry groups has led to an iteration involving an extension of the SL(2,C) Lorentz transformations to arbitrary 2D CKVs.
- Moreover the preferred basis for the action of this symmetry on $\mathcal{S}$-matrix elements is one with definite SL(2,C) weights rather than the standard plane wave basis.



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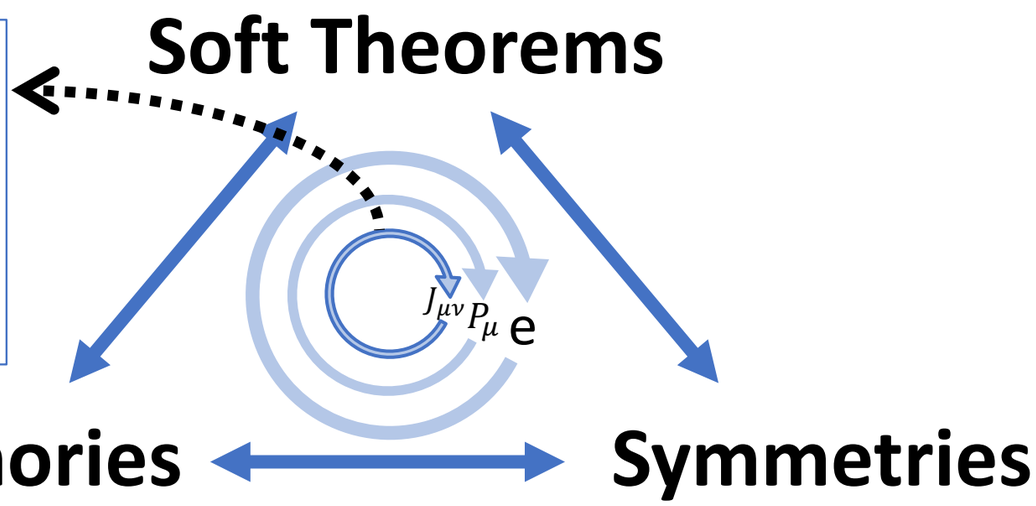
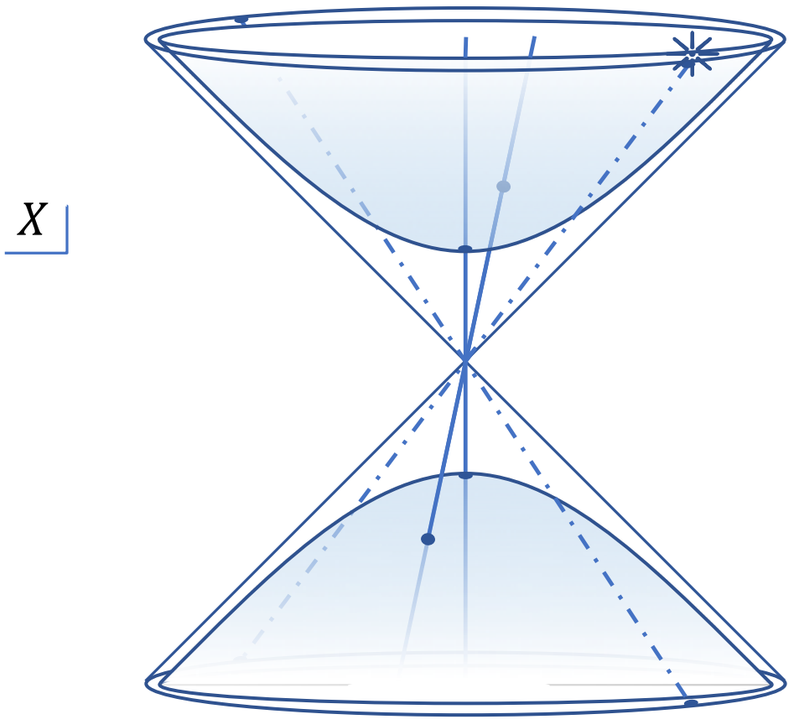
Is a **highest-weight basis** a natural choice for studying implications of superrotations?

- ➢ Recast flat space amplitudes in this form and see what features arise…



Massive Scalars

- Consider the celestial sphere CS^2 as the boundary of the lightcone from the origin in Minkowski spacetime. The projective coordinate w undergoes mobius transformations when the spacetime undergoes Lorentz transformations



$$w = \frac{X^1 + iX^2}{X^0 + X^3} \qquad w \to \frac{aw+b}{cw+d}$$

- We look for solutions to the massive Klein-Gordon equation parameterized by a weight Δ and a reference direction w such that it transforms as a quasi-primary under SL(2,C)

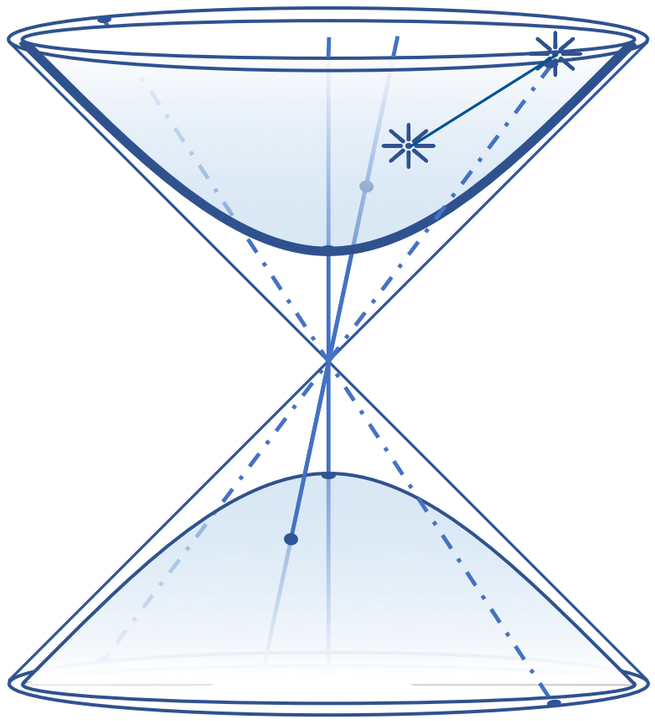
$$\left(\frac{\partial}{\partial X^\nu}\frac{\partial}{\partial X_\nu} - m^2\right)\phi_{\Delta,m}(X^\mu; w, \bar{w}) = 0$$

$$\phi_{\Delta,m}\left(\Lambda^\mu{}_\nu X^\nu; \frac{aw+b}{cw+d}, \frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}\right) = |cw+d|^{2\Delta}\phi_{\Delta,m}\left(X^\mu; w, \bar{w}\right)$$

[arXiv:1701.00049]

Massive Scalars

- If we looked at single hyperbolic slice of constant unit distance from the origin in $\boldsymbol{R}^{1,3}$ described by coordinates:



$$ds^2_{H_3} = \frac{dy^2 + dz d\bar{z}}{y^2}$$

$$\hat{p}^\mu(y, z, \bar{z}) = \left(\frac{1 + y^2 + |z|^2}{2y}, \frac{\mathrm{Re}(z)}{y}, \frac{\mathrm{Im}(z)}{y}, \frac{1 - y^2 - |z|^2}{2y} \right)$$

- We can construct a bulk to boundary propagator that transforms covariantly under SL(2,C)

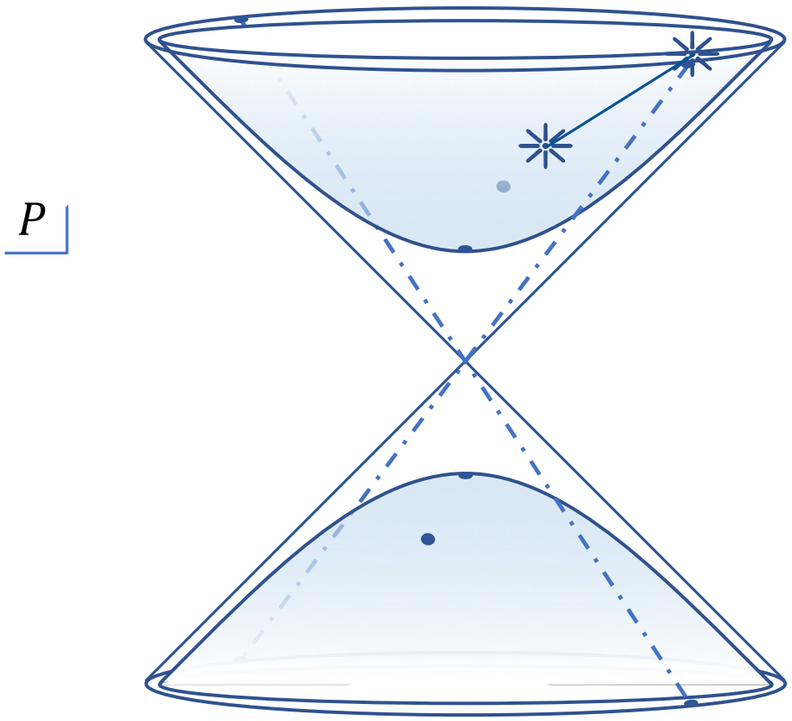
$$G_\Delta(y, z, \bar{z}; w, \bar{w}) = \left(\frac{y}{y^2 + |z - w|^2} \right)^\Delta$$

$$G_\Delta(y', z', \bar{z}'; w', \bar{w}') = |cw + d|^{2\Delta} G_\Delta(y, z, \bar{z}; w, \bar{w})$$

[arXiv:1701.00049]

Massive Scalars

- If we think of (y, z) as coordinates in momentum space, we can use this bulk-to-boundary propagator to construct the desired solutions:



$$\phi^{\pm}_{\Delta,m}(X^\mu; w, \bar{w}) = \int_0^\infty \frac{dy}{y^3} \int dz d\bar{z}\, G_\Delta(y, z, \bar{z}; w, \bar{w})\, \exp\left[\pm i m\, \hat{p}^\mu(y, z, \bar{z})\, X_\mu\right]$$

- Moreover, we can apply this transform directly to the amplitude to convert $\boldsymbol{S}$-matrix elements to a highest weight basis:

$$\tilde{\mathcal{A}}_{\Delta_1,\cdots,\Delta_n}(w_i, \bar{w}_i) \equiv \prod_{i=1}^n \int_0^\infty \frac{dy_i}{y_i^3} \int dz_i d\bar{z}_i\, G_{\Delta_i}(y_i, z_i, \bar{z}_i; w_i, \bar{w}_i)\, \mathcal{A}(m_j \hat{p}_j^\mu)$$

$$\tilde{\mathcal{A}}_{\Delta_1,\cdots,\Delta_n}\left(\frac{a w_i + b}{c w_i + d}, \frac{\bar{a}\bar{w}_i + \bar{b}}{\bar{c}\bar{w}_i + \bar{d}}\right) = \left(\prod_{i=1}^n |c w_i + d|^{2\Delta_i}\right) \tilde{\mathcal{A}}_{\Delta_1,\cdots,\Delta_n}(w_i, \bar{w}_i)$$

[arXiv:1701.00049]

Massive Scalars

- If we look at the Klein-Gordon inner product for two such states with distinct reference vectors

$$(\phi_1^+, \phi_2^+) = -i \int d^3\vec{X} \left[\phi^+_{\Delta_1,m}(X^\mu; w_1, \bar{w}_1) \partial_{X^0} \phi^{+*}_{\Delta_2,m}(X^\mu; w_2, \bar{w}_2) - \partial_{X^0} \phi^+_{\Delta_1,m}(X^\mu; w_1, \bar{w}_1) \phi^{+*}_{\Delta_2,m}(X^\mu; w_2, \bar{w}_2) \right]$$
$$= 2(2\pi)^3 m^{-2} \int_0^\infty \frac{dy}{y^3} \int dz d\bar{z} G_{\Delta_1}(y, z, \bar{z}; w_1, \bar{w}_1) G^*_{\Delta_2}(y, z, \bar{z}; w_2, \bar{w}_2)$$

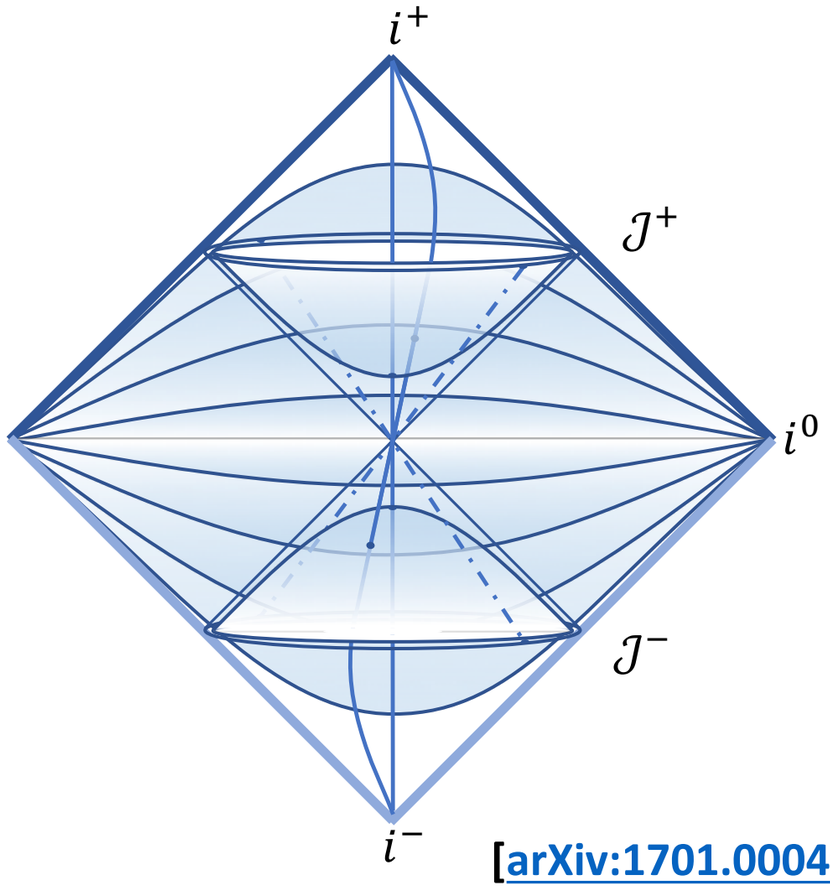
we get an integral that we can make sense of in a distributional manner if we take $\Delta_k = a + i\lambda_k$

$$(\phi_1^+, \phi_2^+) = 64\pi^5 m^{-2} \frac{1}{(\Delta_1 + \Delta_2^* - 2) \left| w_1 - w_2 \right|^{\Delta_1 + \Delta_2^*}} \delta(\lambda_1 + \lambda_2)$$

[arXiv:1701.00049]

Massive Scalars

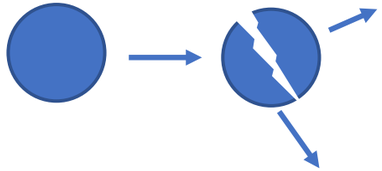
- The way in which 2D conformal symmetry dictates the form of correlation functions is seen here as Lorentz invariance where the physical spacetime is what is typically the abstract embedding space
- Other efforts towards a flat space holographic description [hep-th/0303006,arXiv:1609.00732] have looked at a foliation of Minkowski space to reproduce AdS/CFT, dS/CFT on each slice or convolving m=0 highest weight states in ex. only the forward lightcone region to reproduce CFT-like correlators.

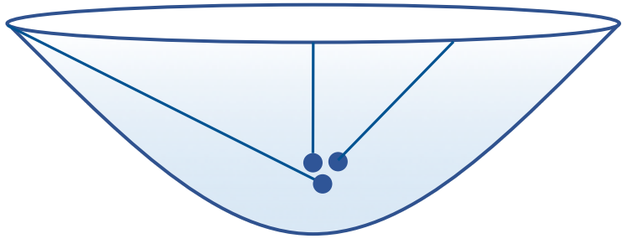


[arXiv:1701.00049]

Massive Scalars

- Here, the bulk to boundary propagator acts on the amplitude in momentum space. As such it probes the amplitude at all energy scales (vs only needing to know matrix elements between low energy states in an effective theory)
- The behavior of low-point "correlation functions" is strongly dictated by momentum conservation in the bulk. Special scattering configurations can be used to get Witten diagram-like results.





$$2(1+\epsilon)m \to m + m$$

$$\tilde{\mathcal{A}}(w_i, \bar{w}_i) = \frac{i2^{\frac{9}{2}}\pi^6\lambda\Gamma(\frac{\Delta_1+\Delta_2+\Delta_3-2}{2})\Gamma(\frac{\Delta_1+\Delta_2-\Delta_3}{2})\Gamma(\frac{\Delta_1-\Delta_2+\Delta_3}{2})\Gamma(\frac{-\Delta_1+\Delta_2+\Delta_3}{2})\sqrt{\epsilon}}{m^4\Gamma(\Delta_1)\Gamma(\Delta_2)\Gamma(\Delta_3)|w_1-w_2|^{\Delta_1+\Delta_2-\Delta_3}|w_2-w_3|^{\Delta_2+\Delta_3-\Delta_1}|w_3-w_1|^{\Delta_3+\Delta_1-\Delta_2}} + \mathcal{O}(\epsilon)$$

[arXiv:1701.00049]

m=0, MHV

- By holding $\omega \equiv \frac{m}{2y}$ fixed as we take m to zero, we can see how the massless limit of our transform can be deduced from the boundary behavior of our momentum space bulk-to-boundary propagator:

$$G_\Delta(y, z, \bar{z}; w, \bar{w}) \sim C y^{2-\Delta} \delta^2(z - w) + \frac{y^\Delta}{|z - w|^{2\Delta}} + \ldots$$

- This motivates defining massless highest weight states in terms of a frequency mellin transform, which obey an orthogonality condition for $\Delta = 1 + i\lambda$ (unitary principal series representation)

$$\mathfrak{a}_\lambda(\hat{q}) \equiv \int_0^\infty d\omega \; \omega^{i\lambda} a(\omega, \hat{q})$$

$$\langle 0 | \mathfrak{a}_{\lambda'}(\hat{q}') \mathfrak{a}^\dagger_\lambda(\hat{q}) | 0 \rangle = 2(2\pi)^4 \delta(\lambda - \lambda') \delta^{(2)}(\hat{q} - \hat{q}')$$

m=0, MHV

- It is curious how this particular choice of weights produces integrals over all energies that are well defined for the tree level MHV amplitudes (at least in a distributional sense):

$$\tilde{\mathcal{A}}_{\Delta_1,\cdots,\Delta_n}(w_i,\bar{w}_i) \equiv \prod_{k=1}^{n}\int_0^\infty d\omega_k \omega_k^{i\lambda_k}\,\mathcal{A}(\omega_k q_k^\mu)$$

$$\langle ij\rangle = 2\sqrt{\omega_i\omega_j}(w_i - w_j)$$

$$\tilde{\mathcal{A}}_n = \prod_{k=1}^{n}\int_0^\infty d\omega_k \omega_k^{i\lambda_k}\,\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}\delta^4(\sum_k p_k)$$

$$\tilde{\mathcal{A}} \propto \int_0^\infty ds s^{i\sum\lambda_k - 1} = 2\pi\delta(\sum\lambda_k)$$

m=0, MHV

- The low point amplitudes are essentially fixed by the 4D kinematics. The MHV 4pt is the first where non-contact configurations are allowed, although a cross ratio constraint remains from the manner in which four null momenta summing to zero are not linearly independent. For a $2 \to 2$ process with helicities $(- - \; + \; +)$

$$\tilde{\mathcal{A}}_4 = (-1)^{1-i\lambda_2+i\lambda_3} \frac{\pi}{2} \left[\frac{\eta^5}{1-\eta} \right]^{1/3} \delta(\mathrm{Im}[\eta])$$

$$\times \, \delta(\lambda_1 + \lambda_2 - \lambda_3 - \lambda_4) \prod_{i<j}^{4} z_{ij}^{h/3-h_i-h_j} \, \bar{z}_{ij}^{\bar{h}/3-\bar{h}_i-\bar{h}_j}$$

$$h_1 = -\frac{i}{2}\lambda_1 \quad \bar{h}_1 = 1 - \frac{i}{2}\lambda_1$$

$$h_2 = -\frac{i}{2}\lambda_2 \quad \bar{h}_2 = 1 - \frac{i}{2}\lambda_2$$

$$h_3 = 1 + \frac{i}{2}\lambda_3 \quad \bar{h}_3 = \frac{i}{2}\lambda_3$$

$$h_4 = 1 + \frac{i}{2}\lambda_4 \quad \bar{h}_4 = \frac{i}{2}\lambda_4$$

m=0, MHV

- Although 3 pt massless scattering restricts the momenta to be collinear, one can go to (2,2) signature where the z are real variables

$$\tilde{\mathcal{A}}_3(\lambda_i; z_i, \bar{z}_i) = \pi(-1)^{i\lambda_1}\mathrm{sgn}(z_{23})\mathrm{sgn}(z_{13})\,\delta(\sum_i \lambda_i)\frac{\delta(\bar{z}_{13})\delta(\bar{z}_{12})}{z_{12}^{-1-i\lambda_3}\,z_{23}^{1-i\lambda_1}\,z_{13}^{1-i\lambda_2}}$$

- And then use BCFW, combined with mellin and inverse mellin transforms to check consistency of the 4 pt result.

m=0, MHV – How far can we take this?

- Curious thoughts: Is there sense in which we could rephrase recursion relations for 4D massless scattering amplitudes, like BCFW, in terms of 2D OPE statements?
- Not there yet, ex. singular behavior for low point functions would seem prohibitive to such an interpretation, but have option combining a shadow, smeared solution:

$$\mathcal{O}^{+}_{i\lambda}(w,\bar{w}) = \phi^{+}_{i\lambda}(w,\bar{w}) + C_{+,\lambda}\int d^2z \frac{1}{(z-w)^{2+i\lambda}(\bar{z}-\bar{w})^{i\lambda}}\phi^{-}_{-i\lambda}(z,\bar{z})$$

$$\mathcal{O}^{-}_{i\lambda}(w,\bar{w}) = \phi^{-}_{i\lambda}(w,\bar{w}) + C_{-,\lambda}\int d^2z \frac{1}{(z-w)^{i\lambda}(\bar{z}-\bar{w})^{2+i\lambda}}\phi^{+}_{-i\lambda}(z,\bar{z})$$

still use MHV mellin amplitudes as building blocks

Flat space amplitudes in a highest-weight basis:

A natural choice for studying implications of superrotations?

SABRINA GONZALEZ PASTERSKI