Physics 211r: Problem Set 5

Due: May 4, 2016

Reading Assignment

[1] S. W. Hawking, M. J. Perry and A. Strominger, "Soft Hair on Black Holes," arXiv:1601.00921 [hep-th].

[2] E. E. Flanagan and D. A. Nichols, arXiv:1510.03386 [hep-th].

[3] S. Pasterski, "Asymptotic Symmetries and Electromagnetic Memory," arXiv:1505.00716 [hep-th].

[4] S. Pasterski, A. Strominger and A. Zhiboedov, "New Gravitational Memories," arXiv:1502.06120 [hep-th].

[5] A. Strominger and A. Zhiboedov, "Gravitational Memory, BMS Supertranslations and Soft Theorems," JHEP **1601**, 086 (2016) doi:10.1007/JHEP01(2016)086 [arXiv:1411.5745 [hep-th]].

1. In this problem, we will study the gravitational memory effect experienced by inertial observers due to supertranslations.

   (a) Working in retarded Bondi coordinates, determine the 4-velocity of an inertial time-like observer (i.e. an observer travelling along a time-like geodesic) in the large-r limit. Determine the u and r components to order $\frac{1}{r}$ and the z and $\overline{z}$ components to order $\frac{1}{r^2}$. Normalize the 4-velocity in the usual way ($v^\mu v_\mu = -1$), and require that at $u = u_0$ the trajectory pass through the point $(r_0, z_0, \overline{z}_0)$ where $r_0 \gg u_0$.

   (b) Now consider a nearby inertial observer who passes through the $(r_1, z_1, \overline{z}_1)$ at $u = u_0$, where $r_1 = r_0 + \delta r$ with $\delta r \ll r_0$. Determine the proper distance between the observers to leading order in δr.

   (c) A burst of radiation of the form

   $$T^M_{uu}(u, r, z, \overline{z}) = \frac{\mu}{4\pi r^2}\delta(u - u_{\rm rad})\delta^2(z - z_{\rm rad}), \qquad u_{\rm rad} > u_0, \qquad \frac{|u_{\rm rad} - u_0|}{r_0} \ll 1 \ , \quad (1)$$

   passes the observers. Determine the change in proper distance between the observers as a result of the passage of this radiation.

   (d) Find the supertranslation f that gives rise to the same change in proper distance.

2. Consider the Vaidya black hole created by a spherically symmetric null shockwave at $v = 0$. We define the horizon large gauge charge as

$$Q_\varepsilon^{\mathcal{H}} \equiv \frac{1}{e^2} \int_{\mathcal{H}^+} \varepsilon \, (*F) \ . \tag{2}$$

where $\mathcal{H}^+$ is the future of the horizon of the Vaidya black hole.

(a) Using the constraints derived from Maxwell's equations on $\mathcal{H}$, rewrite $Q_\varepsilon^{\mathcal{H}}$ as an integral over $\mathcal{H}$ and determine the soft and hard part of this charge. Write the soft charge explicitly in the $(v, r, z, \overline{z})$ coordinates.

(b) Determine the symplectic form for the gauge fields on $\mathcal{H}$. *Hint: You can use the symplectic form Ω_Σ that was derived in Problem Set 1 and set $\Sigma = \mathcal{H}$.* Write it out explicitly in the $(v, r, z, \overline{z})$ coordinates.

(c) By using the explicit forms of the charge and symplectic form derived in parts (a) and (b), show that $Q_\varepsilon^{\mathcal{H}}$ generates large gauge transformations on the horizon, i.e.

$$[Q_\varepsilon^{\mathcal{H}}, A_z\big|_{\mathcal{H}}] = i\partial_z \varepsilon \ . \tag{3}$$

3. Consider a Schwarzschild black hole with mass M perturbed by a null shockwave at $v = v_0$ described by the stress tensor

$$\begin{aligned} T_{vv} &= \frac{1}{4\pi r^2}\left[\mu + \frac{1}{4} D^2\left(D^2 + 2\right) f\right] \delta\left(v - v_0\right) - \frac{3M}{8\pi r^3} D^2 f \delta\left(v - v_0\right) \ , \\ T_{vA} &= -\frac{3M}{8\pi r^2} D_A f \delta\left(v - v_0\right) \ . \end{aligned} \tag{4}$$

(a) Solve the linearized Einstein's equations about the initial Schwarzschild solution in Bondi gauge. Assume that only the zero mode of the Bondi mass aspect on $\mathcal{I}^-$ changes under this perturbation.

(b) Show that the final black hole (after $v > v_0$) can be obtained instead by sending a spherically symmetric null shockwave with stress tensor

$$T_{vv} = \frac{\mu}{4\pi r^2} \delta\left(v - v_0\right) \tag{5}$$

followed by an infinitesimal supertranslation.